Australian and New Zealand Industrial and Applied Mathematics



# ANZIAM 2014

The 50th ANZIAM Conference Rotorua, New Zealand 2–6 February 2014

The abstracts of the talks in this volume were set individually by the authors. Only minor typographical changes have been made by the editors. The opinions, findings, conclusions and recommendations in this book are those of the individual authors.

We are grateful to the organisers of the ANZIAM 2012 conference held in Warrnambool, Victoria for providing the template files that they used for their book.

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The cover image by Eusebius Doedel, Bernd Krauskopf and Hinke Osinga shows one half of the Lorenz manifold together with its intersection curves with a sphere; the famous butterfly attractor is shown inside. The picture is related to Bernd Krauskopf's talk on Thursday.

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## 1 Conference Details and History

## 1.1 Organising Committee

- Steve Taylor (University of Auckland) director and SIAM representative
- Winston Sweatman (Massey University)
- Graeme Wake (Massey University)
- Shixiao Wang (University of Auckland)
- Andrea Raith (University of Auckland)
- Richard Clarke (University of Auckland)

## 1.2 Early Career Workshop Committee

- Alex James (Canterbury University)
- Richard Brown (Massey University)
- Roslyn Hickson (University of Newcastle)

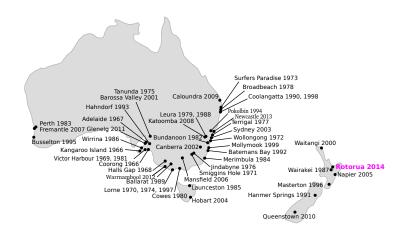
## 1.3 Plenary Speakers Committee

- Robert McKibbin (Massey University) chair
- Graeme Hocking (Murdoch University)
- Vivien Kirk (The University of Auckland)
- Kerry Landman (The University of Melbourne)
- Tim Marchant (University of Wollongong)
- Geoff Mercer (Australian National University)
- Winston Sweatman (Massey University)
- Graham Weir (IRL)
- Ilze Ziedins (The University of Auckland)

## 1.4 Plenary Speakers

- Alison Etheridge (University of Oxford)
- Lisa Fauci (Tulane University)
- Douglas Heggie (University of Edinburgh)
- Shane G. Henderson (Cornell University)
- Shaun Hendy (The University of Auckland) 2013 E.O. Tuck Medallist
- Bernd Krauskopf (The University of Auckland)
- Geoff Mercer (Australian National University) 2013 E.O. Tuck Medallist
- Terence O'Kane (CSIRO) 2013 J.H. Michell Medallist

## 1.5 Past Conference Locations



## 1.6 The T.M. Cherry Student Prize

A student prize was introduced in 1969 at Victor Harbor and is awarded annually for the best student talk presented at the conference. In May 1976, ANZIAM (then the Division of Applied Mathematics) adopted the title "T.M. Cherry Student Prize" in honour of one of Australia's leading scientists, Professor Sir Thomas MacFarland Cherry. Past recipients are listed below.

1969	R. Jones	The University of Adelaide
1970	J. Rickard	University College London
1971	J. Jones	Mount Stromlo
1974	R. P. Oertel	The University of Adelaide
1975	R. E. Robinson	The University of Sydney
1976	J. P. Abbott	Australian National University
1977	J. Finnigan	CSIRO
	S. Bhaskaran	The University of Adelaide
1978	B. Hughes	Australian National University
	P. Robinson	The University of Queensland
1979	J. R. Coleby	The University of Adelaide
	B. Hughes	Australian National University
1980	M. Lukas	Australian National University
1981	A. Plank	The University of New South Wales
1982	G. Fulford	University of Wollongong
	J. Gear	The University of Melbourne
1983	P. Kovesi	The University of Western Australia
1984	A. Kucera	University of Wollongong
	S. Wright	The University of Queensland
1985	G. Fulford	University of Wollongong
	F. Murrell	The University of Melbourne
1986	A. Becker	Monash University
	K. Thalassoudis	The University of Adelaide
1987	M. Rumsewicz	The University of Adelaide
1988	W. Henry	Australian National University
1989	M. Myerscough	University of Oxford
	J. Roberts	The University of Melbourne

1990	J. Best	University of Wollongong
1991	S. K. Lucas	The University of Sydney
	A. Tordesillas	University of Wollongong
1992	S. F. Brown	The University of Sydney
1993	D. Standingford	The University of Adelaide
1994	B. Barnes	Monash University
1995	A. Buryak	Australian National University
1996	A. Gore	The University of Newcastle
	D. Scullen	The University of Adelaide
1997	S. Cummins	Monash University
1998	J. Clark	The University of Sydney
	T. Gourlay	The University of Adelaide
1999	E. Ostrovskaya	Australian National University
2000	C. Reid	Massey University
2001	M. Haese	The University of Adelaide
2002	V. Gubernov	Australian Defence Force Academy
	W. Megill	University of British Columbia/University of Wollongong
2004	K. Mustapha	The University of New South Wales
2005	J. Looker	The University of Melbourne
2006	C. Fricke	The University of Melbourne
2007	S. Harper	Massey University
2008	E. Button	The University of Melbourne
	M. Haythorpe	University of South Australia
2009	S. Cohen	The University of Adelaide
2010	L. Mitchell	The University of Sydney
2011	S. Butler	The University of Sydney
	J. Caffrey	The University of Melbourne
2012	J. Nassios	The University of Melbourne
2013	D. Khoury	The University of New South Wales
	T. Vo	The University of Sydney

## 1.7 The Cherry Ripe Prize

Since 1995 the students have run an alternative competition for the best non-student talk. The past recipients are listed below.

1995	Natashia Boland	The University of Melbourne
1996	Andrew Pullan	The University of Auckland
1997	Neville de Mestre	Bond University
1998	David Stump	The University of Queensland
1999	Mark McGuinness	Victoria University of Wellington
2000	Joseph Monaghan	Monash University
	Andy Philpott	The University of Auckland
2001	Phil Broadbridge	The University of Wollongong
2002	Ernie Tuck	The University of Adelaide
	Larry Forbes	University of Tasmania
2004	Stephen Lucas	University of South Australia
2005	Kerry Landman	The University of Melbourne
2006	Vicky Mak	Deakin University
	James Sneyd	The University of Auckland
2007	Geoffry Mercer	Australian Defence Force Academy
2008	Neville de Mestre	Bond University
2009	Philip Maini	University of Oxford
2010	Larry Forbes	University of Tasmania
2011	Larry Forbes	University of Tasmania
	Darren Crowdy	Imperial College, London
2012	Martin Wechselberger	University of Tasmania
2013	Scott McCue	Queensland University of Technology
	Sheehan Olver	The University of Sydney

## 1.8 The J.H. Michell Medal

The J.H. Michell Medal is awarded to outstanding new researchers who have carried out distinguished research in applied or industrial mathematics, where a significant proportion of the research work has been carried out in Australia or New Zealand. The past recipients are listed below.

1999	Harvinder Sidhu	The University of New South Wales
2000	Antoinette Tordesillas	The University of Melbourne
2001	Nigel Bean	The University of Adelaide
2002	Stephen Lucas	University of South Australia
2004	Mark Nelson	University of Wollongong
2006	Sanjeeva Balasuriya	The University of Sydney
2007	Yvonne Stokes	The University of Adelaide
2008	Carlo Laing	Massey University
2009	Scott McCue	Queensland University of Technology
2011	Frances Kuo	The University of New South Wales
2012	Matthew Simpson	Queensland University of Technology
2013	Terence OKane	CSIRO, Marine and Atmospheric Research

#### 1.9 The E.O. Tuck Medal

In honour of the late Ernest Oliver Tuck, FAustMS, FTSE and FAA, ANZIAM has instituted a midcareer award for outstanding research and distinguished service to the field of Applied Mathematics. The inaugural EO Tuck Medals were presented at ANZIAM 2013.

2013 Shaun Hendy Victoria University of Wellington and Callaghan Innovation Geoffry Mercer Australian National University

#### 1.10 The ANZIAM Medal

The ANZIAM medal is awarded on the basis of research achievements or activities enhancing applied or industrial mathematics and contributions to ANZIAM. The first award was made in 1995. The past recipients are listed below.

1995	Renfrey Potts	The University of Adelaide
1997	Ian Sloan	The University of New South Wales
1999	Ernie Tuck	The University of Adelaide
2001	Charles Pearce	The University of Adelaide
2004	Roger Grimshaw	Loughborough University
2006	Graeme Wake	Massey University
2008	James Hill	University of Wollongong
2010	Bob Anderssen	CSIRO
2012	Robert McKibbin	Massey University

#### 1.11 The A F Pillow Applied Mathematics Top-up Scholarship

The AF Pillow Applied Mathematics Trust offers an annual 'top-up' scholarship to a student holding either an Australian Postgraduate Award (APA) or equivalent award for full-time research in Applied Mathematics leading to the award of a PhD. The aim of the A F Pillow Applied Mathematics Top-up Scholarship is to increase the quantity and quality of postgraduate students in the field of applied mathematics in Australia. The past recipients are listed below.

2009	Christopher Lustri	Queensland University of Technology
2010	Alex Badran	University of Wollongong
2011	Michael Dallaston	Queensland University of Technology
2012	Hayden Tronnolone	University of Adelaide
2013	Lisa Mayo	Queensland University of Technology

#### 1.12 Acknowledgements

The Organising Committiees of the conference and the early career workshop gratefully acknowledge the financial support of the Australian and New Zealand Industrial and Applied Mathematics Society, Fonterra, The New Zealand Mathematical Society, The Australian Mathematical Society, AMSI, CSIRO publishing, Taylor and Francis Publishing, Wakes Scientific Consulting.

The Organising Committee is especially thankful to CSIRO for sponsoring the following students to attend the ANZIAM 2014 conference:

David Arnold	Pouya Baniasadi	Peter Braunsteins
Rachel Bunder	Sophie Calabretto	Chen Chen
Joshua Christie	Luigi Cirocco	Jesse Collis
Adam Ellery	Megan Farquhar	Adrian Grantham
Ava Greenwood	Alexandra Hogan	Michael Jackson
Wang Jin	Stuart Johnston	David Khoury
Daniel Ladiges	Michael Lydeamore	Louise Manitzky
Alexey Martyushev	Jonathan Mitchell	Ellen Muir
Afia Naheed	Meksianis Ndii	Catherine Penington
Ravindra Pethiyagoda	Ignacio Piwonka	Ramya Rachmawati
Christian Reynolds	Reza Roozbahani	Lisa Schultz
Uzma Sehrish	Jesse Sharp	Robyn Stuart
Budi Sunarko	Mingmei Teo	Johnny Thew
Katrina Treloar	Timm Treskatis	Adam Tunney
Rauf Ummul Fahri	Ali Asher Zaidi	

## 2 Conference Events, Venues and Facilities

#### 2.1 Conference Reception

A Welcome reception will be held from **7:30pm on Sunday the 2nd February** at the Millennium's Poolside Restaurant. All registered delegates and their accompanying guests are invited.

#### 2.2 Early Career Workshop

This takes place after the main conference on Friday 7th February. There is a preliminary meeting of this in Millennium 5, 6:30-7:30 pm Sunday 2nd February, just before the Welcome Reception.

#### 2.3 Conference Banquet

The banquet dinner will be held at the **Blue Baths**, with pre-dinner drinks from **7:30pm on the 1st of February**. Tickets are \$115 per person and must be purchased in advance, either via the conference website prior to the start of the conference or at the conference registration desk. The Blue Baths is a short walk from the Millennium.

#### 2.4 Refreshment Breaks and Lunches

Morning and afternoon tea and light refreshments will be available in the Pre-Function area outside the Mokoia Room on Level 2 (the same level as the hotel entrance) rooms. Lunches are included in the registration fee for delegates and their registered guests. They will be available after the last presentations of the morning sessions in the Poolside Restaurant.

#### 2.5 Women in Mathematics Lunch

Nalini Joshi and Bronwyn Hajek have organised this special lunch which will be held in the Nikau Restaurant at lunchtime on Monday 3rd February. Guest Speakers: Lisa Fauci and Alison Etheridge. Delagates will receive an email message with a request to RSVP before the conference.

#### 2.6 Mathematics in Industry

This event is being organised by the NZ branch of ANZIAM and will take place at the conference in the afternoon and early evening of Monday 3rd February. It will comprise a focus session, consisting of approximately four talks, followed by a forum to facilitate discussion among industry representatives and and anyone with an interest in mathematics in industry.

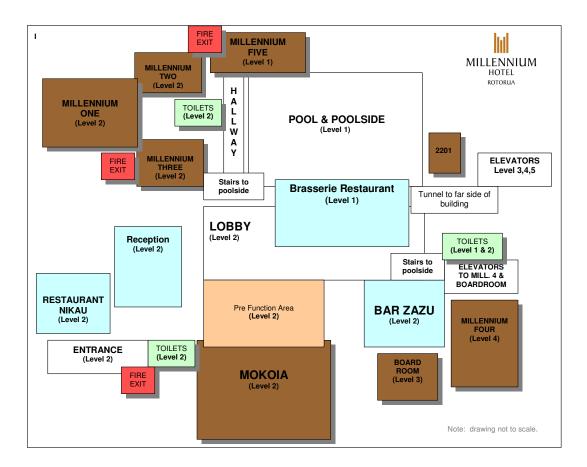
#### 2.7 Internet Access

- Delegates staying at the Millennium as part of the conference package should get 1 GB of internet access per day.
- Free wireless wifi access is available in the the conference area, but it is limited to 20 minutes per day. Tip: If using this, turn wifi off when it is not in use.

• Rotorua's central business district has free wireless internet. This is available in most public places along Tutanekai Street, around the Visitor Centre & i-Site, Village Green and in front of the Museum within the Government Gardens. When in range, your device will see "Rotorua\_FreeWiFi" within its wifi list.

## 2.8 Plenary Lectures and Contributed Talks

All invited plenary lectures will take place in the Mokoia Room. Contributed talks will be held in five parallel sessions in the Mokoia Room and Millennium 1, 2, 4, 5. The duration of each contributed talk will be fifteen minutes with an additional five minutes for questions and room changes.



The Millenium Hotel floor plan (note several floors are shown).

	Monday morning					
		* denotes student talk. Entries in blue indicate changes that occurred after the book was printed.				
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5	
8:00-8:15		Reg	gistration at the ANZIAM of	desk		
8:15 - 8:30			Conference Opening			
8:30-9:30	Chair: Graeme Wake					
	Shaun Hendy Slippery	issues in micro and nanosc	ale flows			
	Chair: Andrea Raith	Chair: Shaun Hendy	Chair: Matthew Simp-	Chair: Tristram Alexan-	Chair: Marianito Ro-	
			son	der	drigo	
9:30-9:50	Jonas Harbering <sup>*</sup> In-	Mike Chen The role of	Kerry Landman Su-	John Mitry* You're As	Christian Reynolds*	
	tegrative Public Trans-	internal pressure in mi-	perstars!	Cold As Ice: A tale of	Modelling the environ-	
	portation Planning with	crostructured optical fi-		multiple timescales	mental impact of weekly	
	LinTim	bre drawing			food consumption in	
					Australia	
9:50-10:10	Rachel Bunder* Pick-	Yvonne Stokes Sinter-	Peter Straka Subdiffu-	Peter Langfield*	Mark Nelson Sludge	
	ing Items for Experi-	ing and stretching in the	sion with Nonlinear Par-	Computing Isochrons	Disintegration in the Ac-	
	mental Sets: Heuristic	drawing of microstruc-	ticle Interaction	as a Boundary Value	tivated Sludge Process	
	methods	tured optical fibres		Problem		
10:10-	Andrew Eberhard A	Jesse F. Collis* Hy-	Stuart Johnston*	Jennifer Creaser*	Noel Barton Air-	
10:30	Scalable Trust Region	drodynamic Trapping	Lattice-free descriptions	alpha-flips and T-points	blown thermal storage	
	Algorithm for Nons-	Using Micro-Vortices	of collective motion with	in the Lorenz system	in pebble beds	
	mooth Optimization		crowding and adhesion			
10:30-	Melih Ozlen Re-	Hayden Tronnolone*	Rachelle Binny* Mul-	Pablo Aguirre Bifur-	Frank de Hoog A Sim-	
10:50	cent advances in	The slumping and	tiscale Modelling of Cell	cations of global invari-	ple Model for Reverse	
	multi objective	stretching under gravity	Invasion	ant manifolds near ho-	Roll Coating	
	mixed/integer/linear	of viscous fluid cylinders		moclinic orbits		
	programming					
10:50-			Morning tea			
11:10						

	Monday morning continued				
	* denotes student talk. Entries in blue indicate changes that occurred after the book was printed.				
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5
	Chair: Rachel Bunder	Chair: Jesse Collis	Chair: Peter Straka	Chair: Jennifer Creaser	Chair: Mark Nelson
11:10-	Ramya Rachmawati*	Sergey A. Suslov Un-	Catherine Pening-	Rua Murray Compu-	Troy Farrell Mathe-
11:30	Crop planning with	expected waves in fer-	ton* Taking interacting	tation of ergodic objects	matical Modelling and
	frequently updated	rofluid convection	motile agents beyond	in dynamical systems	Numerical Simulation of
	weather forecast infor-		monomers and nearest		LiFePO <sub>4</sub> Battery Cath-
	mation		neighbour steps		odes
11:30-	Pouya Baniasadi*	<b>Owen Jepps</b> Density	Louise Manitzky*	Tristram Alexander	Marianito Rodrigo A
11:50	An effective heuristic	dependent models of mi-	Cellular Differentiation	Controlling energy local-	one-compartment model
	algorithm for solving	croporous transport	During Intramembra-	isation in chains of non-	for body cooling and
	the Hamiltonian Cycle		nous Bone Formation:	linear oscillators	time of death estimation
	Problem		A Turing Pattern on a		based only on tempera-
			Growing Domain		ture readings
11:50-	Andrea Raith Robust	Daniel R. Ladiges*	Katrina K. Treloar <sup>*</sup>	Carlo Laing Twisted	Bob Anderssen Re-
12:10	Bicriteria Optimisation	Computational and	Sensitivity of edge detec-	states in phase oscillator	covering information
	in Transport Optimisa-	analytical methods for	tion methods for quan-	arrays	about grain hardness
	tion Problems	gas flows generated by	tifying cell migration as-		from SKCS 4100 mea-
		nanomechanical devices	says		surements
12:10-	David Gao Canoni-	Scott McCue Mathe-	Ali Zaidi* Analysis of	Peter Cudmore*	Ava Greenwood* Un-
12:30	cal Duality and Trial-	matical models for melt-	a stochastic cell growth	Phase and amplitude	derstanding Mild Acid
	ity: Unified Understand-	ing nanoscaled particles	model	dynamics of nonlinearly	Pretreatment of Sugar-
	ing Multi-Scale Complex	need more physics to re-		coupled oscillators	cane Bagasse Through
	Systems and NP-hard	main nice			Mathematical Modelling
	Problems				
12:30-1:50	Lunch in Pools	ide Restaurant	Women in Mathematics Lunch in Nikau Restaurant		
			Chair: Nalini Joshi		
			Al	ison Etheridge and Lisa Fa	uci

		Monday afternoon				
		* denotes student talk. Entries in blue indicate changes that occurred after the book was printed.				
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5	
1:50-2:50	Chair: Kevin Ross, Fonte Shane Henderson Real-		ce Fleets through Statistics	, Simulation and Optimiza	tion	
	Chair: Melih Ozlen	Chair: Daniel Ladiges	Chair: Kerry Landman	Chair: Graeme Wake	Chair: Sarah Lobb	
2:50-3:10	<b>Lynne McArthur</b> Persistence in power output of wind farm time series	<b>Sophie Calabretto*</b> The complex unsteady flow within a fluid- filled annulus and its transition to turbulence	<b>Uzma Sehrish*</b> In Sil- ico Modelling Of Basic Fibroblast Growth Fac- tor Pharmacokinetics In Animal Full Thickness Chronic Wounds	<b>Harvi Sidhu</b> A Tale of Two Flames - Bistability of Combustion Waves	Ignacio Ortega Pi- wonka* Subharmonic Pattern formation and drifting in a quasi one dimensional periodically driven shallow granular bed	
3:10-3:30	Adrian Grantham* Spatial-Temporal Fore- casting of Wind Farm Output	Michael Dallaston Free boundary model of a subglacial meltwater channel cross section	Francis G. Wood- house Predicting the onset of osteoarthritis using discrete dynamical systems	<b>Rhys A. Paul*</b> Travel- ling Waves and Oscilla- tions in Combustion Re- actions	PhuongT.T.Nguyen*Amulti-dimensionalFokker-Planckequationde-scriptionofenergyconversioninmolecularmotors	
3:30-3:50	<b>Andy Philpott</b> Modelling risk in electricity markets	AlisonFrench*Diffusion-driven flows in channels with arbitrarily shaped boundaries	Jesse Sharp* Dis- tinguishing between mean-field, moment dynamics and stochas- tic descriptions of birth-death-movement processes	<b>Robert Marangell</b> A Geometric Approach to Eigenvalues: A New Di- rection in Evans Func- tion Computations	<b>Steve Walters*</b> The Path of a Light Ray: can we Bend it like Einstein?	
3:50-4:10	Luigi Cirocco* Op- timising Revenue for Concentrating Ther- mal Power Plants with Storage Within the Australian National Energy Market	<b>Lisa Mayo*</b> Simulated droplet motion over vir- tual leaf surfaces	Alexander Chalmers* Explor- ing the deformation of the arterial wall: A PDE model for the early stages of atherosclerosis	PetervanHeijsterFrontinteractionsinathree-componentFitzHugh-Nagumosystem	Annette L. Worthy Generation and Control of Solitons using Various Nematic Geometries and Regimes	
4:10-4:30		·	Afternoon tea	•	•	

	Monday afternoon continued					
	* denotes student talk. Entries in blue indicate changes that occurred after the book was printed.					
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5	
	Chair: David Gao	Chair: Daniel Ladiges	Chair: John Murray	Chair: Andy Hammer- lindl	ANZIAM(NZ) In- dustrial Mathematics Session Chair: Boris Baeumer	
4:30-4:50	<b>Anup Purewal*</b> The global effects of Pyragas feedback control	Ravindra Pethiyagoda* Jacobian-free Newton- Krylov methods with GPU acceleration for computing nonlinear ship wave patterns.	<b>Roslyn Hickson</b> The current diabetes epi- demic increases TB transmission	Martin Wechsel- berger Canard Super- sonique	<b>Kevin Ross</b> Analytics at Fonterra and beyond	
4:50-5:10	Helmut Maurer Case Studies of Optimal Con- trol Problems with De- lays	TimmTreskatis*FastSolution of Vis-coplasticFlowProblemswithTrust-RegionSQPMethods	Alexey Martyushev* Modelling the dynamics of immune escape muta- tion in SIV/HIV infec- tion	<b>Theodore Vo*</b> Canards of Folded Saddle-Node Type	<b>Bernd Krauskopf</b> The dynamics of aircraft as ground vehicles	
5:10-5:30	<b>Oliver Zirn</b> Optimal damping of servo axes using Gröbner bases	Simon Clarke Nonlin- ear effects on the fo- cussing of tsunami due to underwater lenses	Matthew Chan* Mod- elling the Spread of a Deliberate Wolbachia Introduction	Kristen Harley* Exis- tence of travelling waves in a model of wound healing angiogenesis	Troy Farrell & Win- ston Sweatman Re- flections on MISG 2014	
5:30-5:50	<b>Johnny Thew*</b> Solv- ing PDE boundary con- trol problems as systems of ODEs.	<b>David Arnold*</b> Exact solutions for thin-film flow in helical channels of arbitrary torsion	Mingmei Teo* Sensi- tivity analysis for the Markovian SIR epidemic model	<b>Graeme Pettet</b> Vascu- lar angiogenic fronts - modelling insights into structure and function	<b>Jo Simpson</b> Determin- ing cheese brining times outcomes from MISG 2014	
5:50-7:20	Mathematics-in-Industry Special Interest Meeting, Millennium 5					
7:40-8:40		Mathematics-in-B	iology Special Interest Mee	ting, Millennium 5		

	Tuesday morning						
	* denotes student talk. Entries in blue indicate changes that occurred after the book was printed.						
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5		
8:30-9:30	Chair: James Sneyd						
	<b>Lisa Fauci</b> Modeling the	bio-fluid dynamics of repr	oduction: successes and cha	allenges			
	Chair: Richard Brown	Chair: Ian Sloan	Chair: Christoph	Chair: Pablo Aguirre	Chair: Alona Ben-Tal		
			Schwab				
9:30-9:50	Richard Clarke Per-	Stephen Joe The	David Horsley* The	Ning Ruan Canonical	Mohd Hafiz Mohd*		
	sistence of hydrodynam-	Hermite normal form	Roots of Bessel Function	Duality Transformation	Modelling the Distri-		
	ics in a two-dimensional	for circulant and skew-	Cross Products	Method for Chaotic	butions of Competing		
	suspension of swimming	circulant lattice rules		Nonlinear Dynamical	Species along Environ-		
	micro-organisms			Systems with Applica-	mental Gradients		
				tions			
9:50-10:10	Andrey Pototsky	Johann Brauchart	Nobutaka Nakazono	Hinke Osinga Com-	Alex James Disen-		
	Rectification of self-	Good point sets on the	The lattice equations	puting failure bound-	tangling nestedness		
	propelled particles by	sphere	arising from the $\tau$ func-	aries by continuation of	from models of complex		
	symmetric barriers		tions of $q$ -P( $A_5^{(1)}$ )	a two-point boundary	ecosystems		
				value problem			
10:10-	Morning tea						
10:30			-				

		Tuesday morning continued					
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5		
	Chair: Richard Clarke	Chair: Stephen Joe	Chair: Annette Worthy	Chair: Carlo Laing	Chair: Bruce van-Brunt		
10:30-	Richard Brown Blood	Yuguang Wang* Rie-	Mikael Slevinsky*	Pingyu Nan* Three	Emily Harvey		
10:50	flow regulation in cere- bral vascular trees	mann localisation on the sphere	New Methods for the Accurate and Efficient Evaluation of Molecu- lar Integrals in the B Function Basis	Time Scale Phenomena	Analysing a mathe- matical model of the division of labour in metabolic pathways		
10:50-	Nazmul Islam* Flow	Kyle Talbot* On an oil	Christoph Schwab	Sebastian Boie*	Michael Plank Smaller		
11:10	through periodic capil- laries	recovery model with sin- gular well data	Rank bounds for Quan- tized Tensor Train (QTT) Approximations of Invariant Distri- butions in Chemical Reaction Networks	Quasi-steady-state re- duction in the analysis of biophysical models	fish to fry?		
11:10-11:30	<b>Pavel Sumets*</b> Mod- elling Post-Capillary Venules	Ian H Sloan Zooming in – Multiscale RBF ap- proximation can be lo- cally refined	RaniaAlharbey*Zero-Absorption Isolinesand ProbeSpectra ofNon-autonomousBlochEquations	<b>Andy Hammerlindl</b> Analysing slow-fast systems using spectral properties	Yuancheng(James)Wang*Modelingmethaneproductionruminants		
11:30-12:30	Chair: Simon Clarke Terry O'Kane The statistical dynamics of geophysical flows with application to ensemble prediction and data assimilation						
12:30-1:30		L	unch in Poolside Restaura	nt			
1:30-7:30	Free time and excursions						
7:30–9:30 9:30–11:30			NZIAM AGM, Millennium M Executive Meeting, Mille				

	Wednesday morning (* denotes student talk)				
			e changes that occurred after		
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5
8:30-9:30	Chair: Ilze Ziedins Alison Etheridge Mode				
	Chair: Galyna Safonova		Chair: Tim Moroney	Chair: Gobert Lee	Chair: Bronwyn Hajek
9:30–9:50	Joshua Christie* Modelling the evolution of uniparental inheri- tance of mitochondria: is mixing bad for the cell?	<b>Robyn Stuart*</b> De- tecting stable regions in the global ocean	John Butcher Ac- curate structure- preserving Runge- Kutta and general linear methods	<b>Geoffrey Decrouez</b> Estimation of the approach rate from border inspection data	Adam Ellery* A sim- plified approach for cal- culating moments of ac- tion for linear reaction- diffusion equations
9:50-10:10	<b>Rowena Ball</b> A new recipe for primordial soup: Hydrogen per- oxide thermochemical oscillator as driver for pre-biotic RNA replication	ShaneKeatingStochasticdatasimilationmethodsderivingsuper-resolvedsatelliteobservationsofthe ocean	<b>Gulshad Imran*</b> Symplectic integrators to effective order methods	<b>David Simpson</b> Effects of Noise on Nonsmooth Dynamical Systems	Matthew Simpson How long does it take for aquifer recharge or aquifer discharge pro- cesses to reach steady state?
10:10– 10:30	<b>Graeme Wake</b> The ghost in our genes - epi- genetics is our new fron- tier	<b>Andrew Keane*</b> Bi- furcation analysis of a model for the El Niño southern oscillation	Awad H. Al-Mohy Exponential Integrators via Single Exponential of a Matrix	<b>Sheehan Olver</b> Nu- merical Random Matrix Theory	<b>Chen Chen*</b> The macroscale boundary conditions for diffusion in a material with microscale varying diffusivities
10:30– 10:50	<b>Peter Kim</b> Modelling evolution of post- menopausal human longevity: The Grand- mother Hypothesis	Jason Cosgrove* In- teracting large-scale at- mospheric vortices	John MacLean* The convergence of higher order schemes for the Projective Integration method for stiff ordinary differential equations	<b>Jie Yen Fan*</b> Mar- tingales with given marginals	<b>Bruce van Brunt</b> An initial-boundary value problem arising in a fragmentation/cell division model
10:50– 11:10			Morning tea		

	Wednesday morning continued (* denotes student talk)				
			e changes that occurred after	÷	
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5
	Chair: Rowena Ball	Chair: Oliver Jensen	Chair: Timm Treskatis	Chair: Ramya Rach-	Chair: Mike Plank
				mawati	
11:10-	Catheryn Gray* Mod-	Robert McKibbin	Qianqian Yang A	Reza Roozbahani*	Karen McCulloch*
11:30	elling the insulin recep-	Instability in sloping	finite volume scheme	Dams Location-	Epidemics on Networks
	tor system for glucose	layered warm-water	with preconditioned	Allocation through	
	transport	aquifers	Lanczos method for	Economic Water Shar-	
			two-dimensional space-	ing	
			fractional reaction-		
			diffusion equaitons		
11:30-	Adelle C. F. Coster	Adam Tunney* A new	Tim Moroney Pre-	Lisa Schultz* Mitigat-	Afia Naheed* Nu-
11:50	Glucose Transporter	mode of instability in	conditioning techniques	ing Traffic-Related Air	merical study of SARS
	Translocation - Inte-	compressible boundary-	for efficient method of	Pollution Exposure in	SEIJTR epidemic model
	grating Models and	layer flows	lines solutions of space-	Adelaide, South Aus-	with the inclusion of
	Experiments		fractional partial differ-	tralia: A Scenario Anal-	diffusion in the system
			ential equations	ysis Using BenMAP	
11:50-	Pengxing Cao* On	W.R.C. Phillips	Megan Farquhar*	Melanie Roberts	David Khoury* Re-
12:10	the roles of modes and	Langmuir circulation in	Exponential Integrators	Streamflow predictions	moval of malaria para-
	stochasticity of inositol	sheared shallow waters	and preconditioned	for industrial applica-	sites by an infected host
	trisphosphate receptors		Krylov Subspace	tions	
	in calcium oscillations in		methods for solving		
	airway smooth muscle		Fractional Diffusion		
	cells		Equations		
12:10-	Shawn Means On or	Stevan Stojanovic*	David Farmer*	Ummul Fahri Abdul	John M Murray
12:30	Off: Simplifying Ion	Homogeneous viscous	Hp-finite element	Rauf* Copula-based	Variability in long-term
	Channel Modeling	flows about corner	methods for the radial-	Approach for Estimat-	HBV dynamics under
		boundaries	dependence of the	ing Return Periods	antiviral therapy
			linearised magnetohy-	of Floods in Victoria,	
			drodynamic equations	Australia	
			in spherical geometries		
12:30-1:30		L	unch in Poolside Restaura	nt	

		Wednesday afternoon (* denotes student talk)						
			e changes that occurred after		٦			
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5			
1:30-2:30		Chair: Winston Sweatman						
	<b>Douglas Heggie</b> Mather	<b>Douglas Heggie</b> Mathematics, Astronomy and Physics a Three-Body Problem						
	Chair: Graham Dono-	Chair: Robert McKibbin	Chair: Megan Farquhar	Chair: Douglas Heggie	Chair: James McCaw			
	van							
2:30-2:50	Jung Min Han* A	Duncan Sutherland*	Quoc Thong Le Gia	Winston Sweatman	Meksianis Ndii* The			
	Mathematical Model of	The dynamics of critical	Higher order QMC	Orbits neighbouring	effect of Wolbachia on			
	Saliva Secretion	points in bounded fluid	Galerkin discretization	equal-mass four-body	dengue transmission			
		flows	for parametric operator	central configurations				
			equations					
2:50 - 3:10	Zoltan Neufeld Pat-	Oliver Jensen Instabil-	James Nichols QMC	Danya Rose* Binary	Alexandra B Hogan*			
	tern formation on the	ities of flexible channel	methods for PDEs with	collisions in the planar 3-	A mathematical model			
	cell cortex	flow	random coefficients	body problem with van-	for respiratory syncytial			
				ishing angular momen-	virus transmission			
				tum				
3:10-3:30	Sophie Hautphenne	Chris Lustri Uncover-	Andrew Stephan*	Attique Ur Rehman*	Michael Lydeamore*			
	Multitype branching	ing water waves with ex-	Generalised compact	Coupled Orbital and	Comparing antiviral al-			
	processes in a random	ponential asymptotics	finite difference meth-	Thermal Evolution of	location strategies			
	environment: Would		ods and the spherical	Major Uranian Satellites				
	they survive forever?		dynamo problem					
3:30 - 3:50	Jonathan Mitchell*	Michael A Page The	Yuto Miyatake* An	Philip W. Sharp	Dylan Lusmore*			
	Despeciation in Phylo-	flow of a rotating fluid at	energy-preserving finite-	Dynamical delivery of	Interfacial dynamics			
	genetics	low Rossby number	difference scheme for the	volatiles to the outer	of computational mod-			
			modified Hunter–Saxton	main belt	els with applications to			
			equation		melanoma and biological			
	<u> </u>			<u> </u>	tissue deformation			
3:50-4:10			Afternoon tea					

	Wednesday afternoon continued (* denotes student talk)					
	Entries in blue indicate changes that occurred after the book was printed.					
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5	
	Chair: Shawn Means	Chair: Michael Dallas- ton	Chair: James Nichols	Chair: Anthony Bedford	Chair: Alexandra Hogan	
4:10-4:30	Alona Ben-Tal New	Inna Elyukhina The	Simon Williams Ad-	Alex Badran* A Mar-	Wang Jin* The devel-	
	insights from modelling central regulation and	flow of non-Newtonian fluids in a vertical oscil-	ditive Dirichlet Models: Mixtures without data	ket Model for VIX Fu- tures	opment of mathemati- cal models to study can-	
	respiratory modulation of heart rate	lating cylinder			cer spheroid growth and drug responsiveness	
4:30-4:50	<b>Nicole Cusimano*</b> Fractional cardiac elec- trophysiological models and novel techniques for their simulation	<b>Graeme Hocking</b> Supercritical withdrawal through a point sink in a porous medium	<b>Peter Johnston</b> A Mathematical Analysis of the "L"–Signal	<b>Peter Braunsteins*</b> Valuing Capacity in Queuing System	<b>Budi Sunarko*</b> Red Blood Cell Classification for Automatic Malaria Diagnosis	
4:50-5:10	Kerry-LynRoberts*RespiratoryRhythmGenerationinthepre-BotzingerComplex	MichaelJackson*Controllingviscousfingering instabilities inHele–Shaw flows	AndrewPapani-colaouFilteringthemaximumlikelihoodformultiscaleproblems	<b>Ellen Muir*</b> Mecha- nism Design and the Trade Problem	Joshua Ross Optimal vaccination strategies	
5:10-5:30	<b>Graham Donovan</b> A dynamic approach to the formation of clustered ventilation defects in the lung	<b>Anis Mohd Alias*</b> The motion of an ob- ject in a viscous strati- fied fluid	Raziyeh Zarredooghabadi* Image registration	Morten Tiedemann* Hold or Sell? - The On- line Knapsack Problem with Incremental Capac- ity	James McCaw Within host influenza infection dynamics: modelling re- infection to understand the role of innate immu- nity	
5:30–5:50		<b>John Shepherd</b> Helical Flow of Yield-stress Flu- ids	<b>Geoff Vasil</b> A connec- tion between ideal mag- netohydrodynamics and nonlinear elastodynam- ics	<b>Babak Abbasi</b> On the Issuing Policies for Per- ishable Items such as Red Blood Cells and Platelets in Blood Ser- vice	MartinaChirilus-BrucknerOnChirilus-BrucknerOnExistenceofBreathersinPeriodicMedia:AnApproachviaInverseSpectralTheory	
7:30pm		Pre-dini	ner drinks and Conference in The Blue Baths	Banquet		

		Thursday morning					
		Entries in blue indicate changes that occurred after the book was printed.					
	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5		
8:30-9:30	Chair: Vivien Kirk	Chair: Vivien Kirk					
	Bernd Krauskopf Disco	overing the geometry of cha	los				
	Chair: Inna Elyukhina	Chair: Alex James	Chair: Peter Johnston	Chair: Alex Badran	Chair: Martina Chirilus-Bruckner		
9:30-9:50	Larry Forbes Can	Claire Postlethwaite	Russell Davies	Peter Taylor A Model	Sarah Lobb Initial		
	Strain and Spin make a	A mathematical model	Wavelet regularization	for the BitCoin Block	value space of inte-		
	Star Bipolar?	of pigeon navigation	for an inverse problem	Chain that takes Prop-	grable lattice systems		
			in linear viscoelasticity	agation Delays into Ac-	and discrete Painlevé		
				count	equations		
9:50-10:10	David Ivers The In-	Galyna Safonova	Gobert Lee Applica-	Song-Ping Zhu Pric-	Yang Shi Integrable		
	compressible Coriolis	Mathematical model of	tion of statistical infer-	ing American-style	quad equations and dis-		
	Operator and Inertial	coupled xylem-phloem	ence in medical images	Parisian options	crete Painlevé equations		
	Modes in Rotating	water transport in leaves					
10.10	Fluids						
10:10-	Peter Miller Weakly	Boris Baeumer Effect	Paul Smith Ap-	Anthony Bedford In-	Bronwyn Hajek		
10:30	Dispersive Internal	of chemokine diffusion	proaches to diffraction	efficiencies, Overround	Nonclassical symmetry		
	Waves	(normal or anomalous)	from impedance coated	and Winning Strategies	solutions for reaction-		
		on leukocytes cytotaxi- city in tissue inflamma-	rectangular structures	in Sport	diffusion equations in		
		tory response			population genetics		
10:30-	Jim Denier Correctly	Jon Borwein Douglas-	Stephen Davis Graph	Guanghua Lian Amer-	Phil Broadbridge In-		
10.50 10:50	model shear-thinning	Rachford Feasibility	Dissimilarity Measures	ican Exchange Options	tegrable radial Burgers		
10.00	fluids	Methods for Matrix	in Biometrics	under Jump-Diffusion	equation in $n$ dimensions		
		Completion Problems		Dynamics			
10:50-		<b>1</b>	Morning tea	v	I		
11:10							

	Thursday morning continued Entries in blue indicate changes that occurred after the book was printed.						
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	Mokoia Room	Millennium 1	Millennium 2	Millennium 4	Millennium 5		
11:10-	Chair: Bob Anderssen						
12:10							
	Geoff Mercer Disease modelling and its impact on policy decisions						
12:10-1:10		Lunch in Poolside Restaurant					

## Plenary

#### Modelling evolution of different genetic types in spatially structured populations.

Alison Etheridge University of Oxford, UK etheridg@stats.ox.ac.uk

Since the pioneering work of Fisher, Haldane and Wright at the beginning of the 20th Century, mathematics has played a central role in theoretical population genetics. One of the outstanding successes is Kingmans coalescent. This process provides a simple and elegant description of the way in which individuals in a population are related to one another. However, it only really applies to very idealised 'unstructured populations in which every individual experiences identical conditions. Spurred on by the need to interpret the recent flood of DNA sequence data, an enormous industry has developed that seeks to extend Kingmans coalescent to incorporate things like variable population size, natural selection and spatial and genetic structure. But, until recently, a satisfactory approach to populations evolving in a spatial continuum has proved surprisingly elusive. In this talk we describe a framework for modelling spatially distributed populations that was introduced in joint work with Nick Barton (IST Austria). As time permits well not only describe the application to genetics, but also some of the intriguing mathematical properties of some of the resulting models.

Modeling the bio-fluid dynamics of reproduction: successes and challenges Lisa Fauci Tulane University in New Orleans, Louisiana fauci@tulane.edu

The process of fertilization in mammalian reproduction provides a rich example of fluid-structure interactions. Spermatozoa encounter complex, non-Newtonian fluid environments as they make their way through the cilialined, contracting conduits of the female reproductive tract. The beat form realized by the flagellum varies tremendously along this journey due to mechanics and biochemical signaling. We will present recent progress on integrative computational models of pumping and swimming in both Newtonian and complex fluids that capture elements of this complex dynamical system.

Mathematics, Astronomy and Physics a Three-Body Problem Douglas Heggie University of Edinburgh, Scotland d.c.heggie@ed.ac.uk

From the time of the ancient Greeks until the death of Poincare just over a hundred years ago, applied mathematics and theoretical astronomy were much the same thing. But the astronomy was celestial mechanics: the motion of the planets. A hundred years ago this was the one part of astronomy which was left almost untouched by the spectacular growth of astrophysics, and the link between mainstream mathematics and mainstream astronomy was severed. The position has been transformed in the last decade or so by the discovery of numerous planetary systems beyond the solar system. It is now a major growth area in astronomy, and has brought the mathematical study of planetary systems once again to centre stage.

In this talk I will focus on the role which the classical gravitational three-body problem is playing in these developments, with particular focus on the averaging technique. It is a rich subject which sheds light on much besides extrasolar planets, including the evolution of stellar clusters around black holes, the death of comets, and even the evolution of the Moon.

#### **Real-Time Control of Ambulance Fleets through Statistics, Simulation and Optimization.** Shane Henderson

School of Operations Research and Information Engineering, Cornell University sgh9@cornell.edu

Ambulance organizations everywhere face increasing call volumes, increasing traffic congestion, and shrinking budgets. To keep response times small, many employ some kind of system-status management (SSM). SSM is the practice of real-time control of the ambulance fleet, using GPS units on the ambulances to track location, and information from the ambulance crews to track status. Available ambulances are carefully stationed in real time, while not requiring too many moves of the ambulance crews. Ill describe recent work in the area, highlighting the use of mathematical tools including statistics, approximate dynamic programming, simulation, and optimization. This work has motivated us to develop simulation optimization algorithms for high-performance computing environments, and Ill discuss what we view as the key complexity there.

Slippery issues in micro and nanoscale flows Shaun Hendy Physics Department, University of Auckland shaun.hendy@auckland.ac.nz

Nano and microscale fluid flows are very important in the natural world and are becoming increasingly useful in a variety of technological settings. Nevertheless there are still many aspects of nanoscale fluid dynamics that are poorly understood. The no-slip boundary condition for the flow of simple liquids over solid surfaces was considered to have been experimentally established in the 1920s, yet new measurement techniques have recently demonstrated that there can be large violations of this boundary condition at the nano and microscale. Here we consider the role of slip boundary conditions in fluid flows using a theoretical approach, complemented by molecular dynamics simulations, and experimental evidence where available. Firstly, we consider nanoscale flows in small capillaries, including carbon nanotubes, where we have observed a breakdown of the Lucas-Washburn equations for capillary uptake. We then consider the general problem of relating macroscopic boundary conditions for fluid flow to microscopic surface chemistry, and discuss several cases where we have been able to solve this problem analytically. Finally, we look at applications of these results to carbon nanotube growth, catalysis, and the best way to put insulation in your roof.

Discovering the geometry of chaos Bernd Krauskopf Mathematics Department, The University of Auckland. b.krauskopf@auckland.ac.nz

The fact that even simple systems can display chaotic dynamics, has been quite a revelation. The Lorenz system of just three ordinary differential equations is arguably the most famous chaotic dynamical system: its chaotic 'butterfly attractor appears on numerous book covers, T-shirts and coffee mugs. But how does the chaos arise from simple dynamics when parameters are changed? This question can be answered by investigating how the three-dimensional phase space is organised by surfaces known as global invariant manifolds, and how this organisation changes on the route to chaotic dynamics. This type of global study is made possible by recently developed numerical methods for finding invariant manifolds that are based on the solution of two-point boundary value problems. On the route to chaos there are intriguing topological and geometrical structures. Did you know, for example, that the chaotic dynamics itself is organised by a space filling pancake?

#### Disease modelling and its impact on policy decisions

Geoff Mercer National Centre for Epidemiology and Population Health, Australian National University. Geoff.Mercer@anu.edu.au

Disease modelling is a powerful tool that can influence government policy and operational decisions but it is currently underutilised. There are three natural time frames of interest to both the modeller and the decision maker: the past, the present and the future. The modelling of past disease outbreaks when coupled with data can provide valuable insights into the mechanisms of disease transmission and inform future interventions. Real time modelling for operational decision support is rarely undertaken but potentially has a huge benefit. The most commonly used disease modelling is future scenario modelling to inform policy decisions such as vaccine rollout. In this talk I will discuss the three different time frames of disease modelling and give case studies of each.

# The statistical dynamics of geophysical flows with application to ensemble prediction and data assimilation

Terry O'Kane CSIRO Marine & Atmospheric Research Terence.O'Kane@csiro.au

Geophysical flows relevant to atmosphere-ocean large-scale circulations are complex, involving the interaction of inhomogenities (mean flows and topography) with turbulent eddies. An understanding of the statistics of turbulent geophysical flows is difficult due to the range of scales encompassed and the complexity of the interactions involved. The number of ensemble members required for direct numerical simulations to adequately sample the probability distribution function is in general prohibitive and until recently the development of a computationally tractable statistical dynamical theory for inhomogeneous turbulent flows has proved elusive. In this talk I will describe recent advances in the non-equilibrium statistical theory and numerics for geophysical flows and their application to problems in data assimilation and weather prediction.

## Normal Sessions

## On the Issuing Policies for Perishable Items such as Red Blood Cells and Platelets in Blood Service

Babak Abbasi School of Mathematical and Geospatial Sciences, RMIT University, Melbourne, Australia babak.abbasi@rmit.edu.au

Red blood cells (RBCs) and platelets are examples of perishable items with a fixed shelf life. Recent studies show that transfusing fresh RBCs may lead to an improvement of patient outcomes. In addition, to better manage their inventory, hospitals prefer to receive fresh RBCs and platelets. Therefore, as well as minimizing outdates and shortages, reducing the average age of issue is a key performance criterion for blood banks. The issuing policy in a perishable inventory system has a substantial impact on the age of issue and outdate and shortage rates. Although several studies have compared the last in first out (LIFO) and the first in first out (FIFO) policies for perishable products, only a few studies have considered the situation of blood banks where replenishment is not controllable. In this study, we examine various issuing policies for a perishable inventory system with uncontrollable replenishment, and outline a modified FIFO policy. Our proposed modified FIFO policy partitions the inventory into two parts such that the first part holds the items with age less than a threshold. It then applies the FIFO policy in each part and the LIFO policy between the parts. We present two approximation techniques to estimate the average age of issue, the average time between successive outdates and the average time between successive shortages of the modified FIFO policy. Our analysis shows in several cases that where the objective function is a single economic function, or it is formulated as a multi-objective model, the modified FIFO policy outperforms the FIFO and LIFO policies.

Bifurcations of global invariant manifolds near homoclinic orbits Pablo Aguirre Universidad Técnica Federico Santa María (Chile) pablo.aguirre@usm.cl Coauthors: Bernd Krauskopf and Hinke Osinga

This work is concerned with the global organization of the dynamics near a homoclinic orbit of a saddle equilibrium of a vector field in  $\mathbb{R}^3$ . If the saddle equilibrium has two real (stable) eigenvalues, then the homoclinic orbit converges generically to the saddle along the direction given by the weak stable eigenvector. On the other hand, if the equilibrium has a complex pair of (stable) eigenvalues, it is a saddle-focus, and the homoclinic orbit converges toward the equilibrium in a spiralling fashion. The transition from a saddle to saddle-focus homoclinic bifurcation occurs at a so-called Belyakov point. In this talk, we focus on the case when the unfolding of the Belyakov point leads to the creation of a single stable periodic orbit. In this setting, we describe the possible configurations of global invariant manifolds near this codimension-two Belyakov homoclinic point in a model by Sandstede. In this way, we shed light on the organization of basins of attractions and explain the appearance of countably many connecting orbits between the saddle and a second equilibrium.

Exponential Integrators via Single Exponential of a Matrix

Awad H. Al-Mohy King Khalid University, Abha, Saudi Arabia ahalmohy@kku.edu.sa Coauthors: Nicholas J. Higham

Many semidiscretized partial differential equations naturally take the form  $dy/dt = Ay + f(t, y), y(0) = y_0$ , where y is  $n \times 1$  vector and A is  $n \times n$  matrix representing a spatially discretized linear operator and f(t, y)containing the nonlinear terms. Exponential integrators are a broad class of methods for the above ODE that treat the linear term exactly and integrate the remaining part of the solution numerically, using an explicit scheme. In this talk we will show that the sums of the form  $\sum_{k=0}^{p} g_k(A)u_k$  that arise in exponential integrators, where the  $g_k$  are related to the exponential function, can be expressed in terms of a single exponential of a matrix of dimension n+p built by augmenting A with additional rows and columns. Then we will describe a new algorithm for computing  $e^{tA}B$ , where A is an  $n \times n$  matrix and B is  $n \times n_0$  with  $n_0 \ll n$ . The algorithm works for any A, its computational cost is dominated by the formation of products of A with  $n \times n_0$  matrices, and the only input parameter is a backward error tolerance. Our numerical experiments show that the algorithm performs in a numerically stable fashion across a wide range of problems.

#### Controlling energy localisation in chains of nonlinear oscillators

Tristram Alexander School of PEMS, UNSW Canberra, Australia 2610 T.Alexander@adfa.edu.au

Nonlinear oscillators are pervasive, from the obvious pendulums and springs of mechanical systems to more stylised situations where they provide a convenient basis for complex processes such as friction. Beneath seemingly different physical scenarios there are often universal models which allow investigation of features common across the different cases. In this work we explore the effect on the dynamics of different types of driving, the new behaviour which emerges with increased dimensionality, and the role of defects. We focus predominantly on the seemingly simple case of a horizontally shaken pendulum chain, and its closely related generic model, the ac-driven damped Frenkel Kontorova model. We identify the localised energy states, known as discrete breathers, and examine their connection to more fundamental periodic solutions. We reveal a means to generate these states through instability and show that unexpected subharmonic excitations may also occur in these chains and form the basis for energy localisation. Finally we explore the effect of defects on energy localisation and pattern formation and find new features emerge with increasing chain length. We consider the generalisation of our results to a higher dimensional system, a chain of coupled spring pendulums, and find similar pattern formation and energy localisation effects.

#### Zero-Absorption Isolines and Probe Spectra of Non-autonomous Bloch Equations Rania Alharbey King Abdulaziz University, Mathematics Department, Faculty of Science. Jeddah, Saudi Arabia rallehabi@kau.edu.sa Coauthors: S. S. Hassan

Probing a physical system by a weak pulse provides two specific benefits within the context of quantum information processing and quantum computing: one, is gain or loss of energy by the pulse from the system. Two, is control of the pulse propagation speed inside the medium. Here, we investigate, mathematically and computationally, the absorption-dispersion spectra of a weak signal field probing a system of 2-level atom coherently driven by a non-resonant mono- or bi-chromatic laser field of arbitrary strength and damped by a non-resonant squeezed vacuum radiation reservoir. Regions of zero-absorption accompanied by finite speed are specifically identified.

#### The motion of an object in a viscous stratified fluid Anis Mohd Alias Monash University anis.alias@monash.edu

This research considers the behaviour of an object, in particular a sphere, immersed in an unbounded viscous stratified fluid. One might expect that there is no fluid motion when both sphere and fluid are at stationary. However, it is observed that a very slow fluid motion is induced by pressure, as well as density, differences in the fluid. This project involves finding analytical solutions of streamfunction and density by the method of matched asymptotic expansions. Both "inner" and outer solutions are obtained by solving the Navier-Stokes equation with corresponding boundary conditions. The differential equations for inner solution are solved using a standard approach with a Fourier Transform method for the outer solution. The outer solution involves the Bessel function of which the inverse Fourier Transform is difficult to find. Therefore, the equivalent problem for a stationary cylinder is considered in order to gain a better understanding of sphere problem. Current result of this project shows that the approximated analytical solution of stationary cylinder exists.

#### References:

[1] Allshouse, M.A., Barad, M.F., & Peacock, T. (2010). Propulsion generated by diffusion-driven flow. Nature Physics, 6, 516-519. doi:10.1038/nphys1689

[2] Van Dyke, M. (1975). Perturbation Methods in Fluid Mechanics. Stanford, CA: Parabolic Pr.

[3] Zvirin, Y., & Chadwick, R.S. (1975). Settling an axially symmetric body in a viscous stratified fluid. International Journal of Multiphase Flow, 1, 743-752. doi:10.1016/0301-9322(75)90033-6

Recovering information about grain hardness from SKCS 4100 measurements Bob Anderssen CSIRO Computational Informatics Bob.Anderssen@csiro.au

A key requirement for efficient plant breeding is to have good (quantitative) phenotypes which allow different genotypes to be quick and easily classified into representative equivalence classes. For wheat, grain hardness is playing an increasingly important role.

On an SKCS 4100 device, the response of an individual grain to its compression can be measured as an individual crush response profile (iCRP). Though there is considerable variability in the iCRPs for a set of grains from the same variety, it appears to be stochastic since, for sufficiently large numbers of randomly chosen grains of the same wheat variety, the averaging of their iCRPs consistently gives the same average crush response profile (aCRP). The advantage of the aCRPs is that (i) they are rheological encapsulations of the dynamics of the fractioning of individual wheat grains under compression, and (ii) the rheological features in their double hump structure can be used to define rheological phenotype phases (RPPs).

The talk will explain how the RPPs identify clear differences in the biological structure of hexaploid and durum wheats. There is a strong connection to fundamental aspects of mathematics through the rheology which will be discussed.

#### Exact solutions for thin-film flow in helical channels of arbitrary torsion

David Arnold The University of Adelaide david.arnold@student.adelaide.edu.au Coauthors: Yvonne Stokes (The University of Adelaide), Ed Green (The University of Adelaide)

The study of fluid flow in helical channels is relevant to applications as diverse as spiral particle separators used in the mining industry and the flow of perilymph fluid in the human cochlea. In this presentation we will discuss a model for steady, fully-developed laminar flow in helical channels of arbitrary torsion and curvature. Since flows in spiral particle separators are typically shallow relative to their width, we take the thin-film approximation to simplify the governing equations. In this case we will see that we can solve analytically to find expressions for the free-surface profile, fluid velocity components and pressure.

A Market Model for VIX Futures Alex Badran University of Sydney alexjbadran@gmail.com

A new modelling approach that directly prescribes dynamics to the term-structure of VIX futures is introduced in this talk. The approach is motivated by practices observed in interest-rate modelling and the tractability enjoyed by models that directly prescribe dynamics to the VIX. The main contribution is a set of conditions that ensure that the joint market of VIX and equity derivatives remains free from arbitrage when VIX futures are modelled directly. The talk is concluded with an application of the modelling approach and implications regarding the relationship between VIX and index option call surfaces.

#### Effect of chemokine diffusion (normal or anomalous) on leukocytes cytotaxicity in tissue inflammatory response Boris Baeumer University of Otago

bbaeumer@maths.otago.ac.nz Coauthors: Aidin Jalilzadeh, Josie Athens

The inflammatory response of tissue to injury or invasion is a complex phenomenon involving, vascular, humoural and cellular factors. In this work we have developed a spatio-temporal mathematical model consisting of a system of diffusion-advection-reaction equations to capture some aspects of tissue inflammatory response. Our proposed model accounts for antigen recognition followed by the effectors function (activation/regulation/inhibition) and further recruitment of leukocytes from blood, which results in the elimination of antigen and resolution of the infection and returning the tissue environment back to homeostasis. We have been able to simulate real scenarios such as the resolution of acute infection as well as recurring/chronic events.

Leukocytes flux consists of two components: random and tactic (directional). Directional movement of leukocytes in response to chemical gradient (chemotaxis) is the major cytotaxic effect which leads the cells towards the site of injury/infection. The average cell velocity depends on local chemical concentration as well as relative gradient. Non- Gaussian diffusion of chemokine which is modelled via fractional Laplacian of order  $\frac{\alpha}{2}$  with  $0 < \alpha \leq 2$  will result in longer distribution tails than Gaussian. We have investigated the consequences of normal and anomalous diffusion of chemokine on the cells tactic movement, which in turn, will also affect the antigen clearance rate.

3-D Matlab simulations helped us to visualise the movement patterns of cells and diffusion of chemokine. Matlabs FEM suite was used to solve our system of quasilinear parabolic equations.

## A new recipe for primordial soup: Hydrogen peroxide thermochemical oscillator as driver for pre-biotic RNA replication

Rowena Ball *The Australian National University* Rowena.Ball@anu.edu.au Coauthors: John Brindley

IN THE beginning, in the primordial soup that perfused through rock pores near hydrothermal vents, there were no cells and no protein enzymes. The 'RNA world' hypothesis holds that cell-free RNA communities grew on solid surfaces and replicated, before the evolution of DNA. What energy source may have driven RNA replication in such an environment? It seems that thermal cycling would be required to drive RNA replication in the absence of cellular (or other) machinery, because heat is required to dissociate double-stranded or multiplex RNA and a cooler phase is necessary for replication and annealing, given a supply of substrate template. This fact is often overlooked in hypotheses about the origin of life. This presentation describes a previously unrecognised mechanism for driving a replicating molecular system on the prebiotic earth. It is proposed that cell-free RNA replication in the primordial soup may have been driven by spontaneously arising, self-sustained oscillatory thermochemical reactions. To test this hypothesis a well-characterised hydrogen peroxide oscillator was chosen as the driver and complementary RNA strands with known association and melting kinetics were used as the substrate. An open flow system model for the self-consistent, coupled evolution of the temperature and concentrations in a simple autocatalytic scheme is solved numerically, and it is shown that thermochemical cycling drives replication of the RNA strands.

An effective heuristic algorithm for solving the Hamiltonian Cycle Problem Pouya Baniasadi *Flinders University* pouya.baniasadi@flinders.edu.au Coauthors: Pouya Baniasadi, Vladimir Ejov, Jerzy A. Filar, Michael Haythorpe, Serguei Rossomakhine

We introduce a new deterministic, polynomial complexity algorithm for solving the Hamiltonian Cycle Problem (HCP) in undirected graphs of order n, called Snakes and Ladders Heuristic (SLH). We have observed that the algorithm is remarkably successful even in cases where Hamiltonian cycles in the graph are extremely rare, although theoretically the algorithm does not guarantee finding a Hamiltonian Cycle. Initially, all vertices of the graph are placed on a given circle in some order. The graphs edges are classified as either snakes or ladders, with snakes forming arcs of the circle and ladders forming its chords. The heuristic strives to place exactly n snakes on the circle, thereby forming a Hamiltonian cycle. SLH uses transformations inspired by k-opt algorithms such as the, now classical, Lin-Kernighan heuristic to reorder the vertices on the circle in order to transform some ladders into snakes and vice versa. The use of a suitable stopping criterion ensures the heuristic terminates in polynomial time if no improvement is made in  $n^3$  major iterations. Comparison of SLH performance to the state of the art TSP algorithms Concorde and LKH (adapted to solving HCP) is provided. Practical on-line demonstration of SLH will accompany the presentation.

Air-blown thermal storage in pebble beds Noel Barton Sunoba Pty Ltd noel.barton@sunoba.com.au

Pebble beds offer a cheap, durable and efficient way to store solar thermal energy, especially when air is the heat transfer fluid. A simulation code for the design of such storage systems will be described. In the interstices, it is required to solve one-dimensional time-dependent PDEs for conservation of air mass and energy. These are coupled to another PDE for radial heat diffusion in pebbles. The mass and energy equations are solved by a finite volume technique, whilst heat transfer in pebbles is solved by finite differences. An important feature of the results is the sharpness of the interface between hot and cold regions as the bed is charged or discharged.

Key parameters are the depth of the bed, the air flow-rate, the pebble-air heat transfer coefficient and material properties of the pebbles. Results will be shown for various applications and charging regimes.

Inefficiencies, Overround and Winning Strategies in Sport Anthony Bedford *RMIT University* anthony.bedford@rmit.edu.au

In this research, we assess the contracting overround in a variety of sports markets, including golf, NBA, tennis and EPL, over the last ten years. Interestingly, we note the influence of time to start, volume, and round effects that relate to markets offered. We consider a number of modelling approaches, including conditional probabilistic simulation methods, Markov approaches, and modified Elo methods, that can be shown to be effective in prediction. We also outline examples of market manipulation that occur in smaller markets where there is often influence from large investors.

New insights from modelling central regulation and respiratory modulation of heart rate. Alona Ben-Tal Massey University a.ben-tal@massey.ac.nz Coauthors: Sophie S. Shamailov (Massey University) and Julian F.R. Paton (University of Bristol)

We are interested in the mechanisms that give rise to respiratory sinus arrhythmia (RSA) - a heart rate variability at the frequency of breathing. It is widely accepted that the loss of RSA is a prognostic indicator for cardiovascular disease and that the prominent presence of RSA indicates a healthy cardiac system, yet the reasons for this are still being debated. To study these questions, we have developed a minimal model for the neural control of heart rate in mammals. The model takes into account variation in heart rate due to changes in blood pressure, lung volume and neural respiratory drive and can mimic a range of experimental observations including key features of RSA. For example, the model can mimic the growth in the amplitude of RSA with decreasing respiratory frequency which then decreases at frequencies below 7 breaths per minute (for humans). We show that the peak in the frequency response is due to the blood pressure feedback (we show this both numerically and analytically with a linear baroreflex). This and other new insights the model provides extend our understanding of central regulation of the heart rate.

Multiscale Modelling of Cell Invasion Rachelle Binny University of Canterbury rachelle.binny@pg.canterbury.ac.nz

The ability of cells to invade to local sites or distant parts of the body is fundamental for a number of physiological processes, including embryonic development and wound healing. Disruption of the regulatory mechanisms controlling invasion can contribute to pathologies such as vascular disease or cancer. Understanding how interactions occurring at the level of individual cells contribute to the large-scale dynamics of an invading cell population is an important step towards the development of novel therapeutic strategies. We use random walk theory to design a mathematical model which captures the salient features of invasion at an individual level, for example cell migration and proliferation. Our model is constructed using a lattice-free framework where cells are represented as individual agents that can wander freely across a continuous domain, rather than being constrained to discrete lattice sites. We then use spatial moment theory to relate these microscopic events to the emergent behaviour of the whole cell population.

#### Quasi-steady-state reduction in the analysis of biophysical models

Sebastian Boie University of Auckland s.boie@auckland.ac.nz Coauthors: Vivien Kirk and James Sneyd

Many biological systems have the property that some processes evolve much faster than others. Mathematical models of such systems often possess different time scales. A common first step in the analysis of these models is to remove fast variables by making quasi-steady-state assumptions. Unfortunately, quasi-steady-state reduction can sometimes cause significant changes in dynamics. We discuss progress on establishing conditions under which quasi-steady-state reduction is mathematically justifiable and will not disrupt the dynamics of the model.

#### Douglas-Rachford Feasibility Methods for Matrix Completion Problems

Jon Borwein CARMA, The University of Newcastle jon.borwein@gmail.com Coauthors: Fran Aragón and Matthew Tam

Many successful non-convex applications of the Douglas-Rachford method for finding a point in the intersection of two sets can be viewed as the reconstruction of a matrix, with known properties, from a subset of its entries. In this talk we discuss recent successful applications of the method to a variety of (real) matrix reconstruction problems, both convex and non-convex. These include reconstructing proteins, solving Sudoku and finding Hadamard matrices.

#### Good point sets on the sphere

Johann Brauchart University of New South Wales j.brauchart@unsw.edu.au

Good point sets on the sphere (in 3 dimensions or higher) have many applications. A typical application is numerical integration of a function over the sphere using the average of function values at well-chosen points (Quasi Monte Carlo [QMC] method). So-called low-discrepancy configurations are a good choice. But one can do better! Employing reproducing kernel Hilbert space techniques, we derive in a very elegant way a theoretical framework centered at the worst-case numerical integration error of the QMC method. This approach leads to the introduction of "QMC design sequences". Each such sequence has a "strength" that is the essentially largest smoothness index of the Sobolev space from which the integrated functions are taken. QMC methods based on QMC designs achieve (asymptotically) optimal order worst-case error but are not required to be exact for polynomials like spherical designs. (But note that a sequence of spherical designs has strength  $\infty$ .) The worstcase error can take a simple closed form (sum of generalized distances) and at the same time is closely related to an L<sub>2</sub>-discrepancy based on spherical caps. Thus we have an easy to compute criterion which can be used to compare sequences of point sets of different families. Moreover, this criterion can be used to "construct" optimal configurations via optimization. Even "bad" configurations like random points may be usable integration nodes with small integration error when weights are added (weighted QMC methods).

We present numerical results for several families of point sets on the sphere including the explicit construction "Spherical Fibonacci points." The latter has been successfully used for computing "illumination integrals" in computer graphics.

The talk is based on joint works with Josef Dick, Ian Sloan, Yu Guang Wang, and Rob Womersley from UNSW and Ed Saff from Vanderbilt.

#### Valuing Capacity in Queuing Systems

Peter Braunsteins The University of Melbourne petertb@student.unimelb.edu.au

There are many situations where one might wish to place a value on the amount of capacity this is allocated to a finite capacity queue. Such a valuation is especially important when units of capacity in the queue can be bought or sold. In this talk we consider a single server queue run by a manager who has the option of buying and selling capacity at specific moments in time. The manager must decide how much capacity to buy or sell, and the price that should be accepted to ensure that the queueing system generates the maximum expected profit.

Integrable radial Burgers equation in n dimensions. Phil Broadbridge La Trobe University P.Broadbridge@latrobe.edu.au Coauthors: Dimetre Triadis

The one-dimensional Burgers equation is the best known and most often used integrable nonlinear parabolic equation. Its structure resembles that of the incompressible Navier-Stokes momentum equation. However, if the dependent variable u is velocity, a one-dimensional incompressible velocity can only be constant, so the Burgers equation does not have that use. Instead it is used to indicate the consequence of non-vanishing viscosity in compressible gas dynamics. Remarkably, the Burgers travelling wave has a rate of entropy increase that is independent of viscosity, and therefore it applies in the inviscid limit to the Riemann shock wave. It is well-known but not true that none of this can be done in n dimensions. Every gradient solution of Burgers' equation in n dimensions, and therefore every irrotational solution in 3 dimensions, can be found by transforming to a linear diffusion equation. For example, the radial Burgers equation is equivalent to the radial linear diffusion equation. Some applicable solutions will be constructed and discussed.

Blood flow regulation in cerebral vascular trees Richard Brown Massey University R.G.Brown@massey.ac.nz

The brain has the ability to very tightly regulate the supply of nutrients via blood flow under significantly varying changes in metabolic activity and blood pressure. It can respond to highly localised spatial signals, being sensitive even to stimulus of a single neuron. Blood flow is controlled by adjusting the resistance to flow of the small arteries by causing them to constrict or dilate.

No blood vessel is independent from the others however, and what is happening elsewhere in the same vascular tree can effect the ability of a single vessel to control local flow. By considering a simplified, but large-scale, spatially-embedded dynamical model of a vascular tree, we begin to study some of ways in which the geometry of the vascular tree and its spatial embedding affect the ability of the tree to regulate flow.

# An initial-boundary value problem arising in a fragmentation/cell division model

Bruce van Brunt Massey University, Palmerston North b.vanbrunt@massey.ac.nz Coauthors: Adel Almalki

Initial-boundary value problems involving a fragmentation type equation arise in various applications including models for cell division. It can be shown, for a broad class of division kernels, that the general solutions to these problems are asymptotic, as time goes to infinity, to separable solutions known as steady size distributions in the cell growth model. In this talk we look at a special case with a particularly simple division kernel that nonetheless illustrates the role of the separable solutions and the asymptotics. The solution can be found analytically for this choice of kernel. We show that this problem has a continuous spectrum, and that, under certain spatial decay conditions, the general solution is asymptotic to the eigenfunction corresponding to the smallest eigenvalue in the spectrum.

# Picking Items for Experimental Sets: Heuristic methods Rachel Bunder University of Newcastle Rachel.Bunder@uon.edu.au Coauthors: Natashia Boland, Andrew Heathcote

Psychologists and other experiment designers are often faced with the task of creating sets of items to be used in factorial experiments. These sets need to be as similar as possible to each other in terms of the items' given attributes. Previously, we have developed a mixed integer program that solves this problem. However, this approach struggles when solving more complex problems, such as those with hundreds of attributes. In this talk we will explore a variety of heuristic methods to solve this problem, including some using projection algorithms such as Douglas-Rachford.

# Accurate structure-preserving Runge-Kutta and general linear methods John Butcher University of Auckland butcher@math.auckland.ac.nz

The accuracy of a numerical method is closely related to the "order" of the local truncation error. This is a deceptively simple idea for Runge–Kutta methods and requires careful analysis for multivalue or "general linear methods". In the context of this talk, structure preservation will mean the conservation of quadratic constraints. For Runge–Kutta methods, this is exactly what symplectic methods do, but for general linear methods, G-symplectic behaviour is required. For classical methods, numerical order is defined in terms of rooted trees but for symplectic and G-symplectic methods, (un-rooted) trees have the more central role. This talk will survey the relationship between order and symplecticity.

# The complex unsteady flow within a fluid-filled annulus and its transition to turbulence

Sophie Calabretto University of Auckland s.calabretto@auckland.ac.nz Coauthors: Jim Denier (University of Auckland) Trent Mattner (The University of Adelaide) Richard Clarke (University of Auckland)

The analysis of turbulence in transient flows has applications across a broad range of fields. The flow of fluid in an annular container is a paradigm for studying the complex dynamics in these transient flows. We consider the 'spin-up' problem, in which the annular container is spun up from rest to a constant rate of rotation. This approach allows us to examine the development of an impulsively generated axisymmetric boundary layer adjacent to the interior annular wall, its subsequent instability and the larger scale transient features within this class of flows. We examine the small-time boundary layer structure by seeking a solution in terms of Rayleighlayer coordinates, matching the horizontal and vertical solutions by means of a corner Rayleigh layer that is symmetric across the corner bisector. Utilising a quadrilateral spectral element DNS code, ideal for solving problems in cylindrical coordinates systems, we will also describe the boundary-layer development, including the initial ejection and subsequent instability, as well as travelling waves in the Ekman layers which arise at the upper and lower boundaries of the container, and the vortex shedding which occurs in the side-wall Rayleigh layers for higher Reynolds numbers.

### On the roles of modes and stochasticity of inositol trisphosphate receptors in calcium oscillations in airway smooth muscle cells

Pengxing Cao University of Auckland pcao002@aucklanduni.ac.nz Coauthors: Xiahui Tan, Martin Falcke, Graham Donovan, Michael J. Sanderson, James Sneyd

Calcium models with deterministic formulations of the inositol trisphosphate receptor (IP3R) have been used to explain and predict relevant physiological processes for over 20 years, but their unrealistic assumptions were recently challenged by experimental data and stochastic simulations, and so were their validities for model predictions. In this talk, by comparing the deterministic and stochastic modeling approaches, we investigate the role of stochasticity of IP3R in airway smooth muscle calcium oscillations. We find that the termination of each oscillation is primarily modulated by a rapid calcium-inhibition in a deterministic way, whereas the regenerations of oscillations are significantly affected by stochastic openings of IP3Rs, which could be a source of failure for the deterministic models to predict a correct frequency change. In addition, recent IP3R models fitted to singlechannel data in vivo reveal the existence of different modes in which IP3R behaves quite differently, but the role of the modes in modulating calcium oscillations has not been well understood. We will show that the complexity of each mode is not necessary for producing calcium oscillations, but instead the inter-mode transitions evolving on different time scales play a crucial role in this process.

# Exploring the deformation of the arterial wall: A PDE model for the early stages of atherosclerosis Alexander Chalmers The University of Sydney acha5264@uni.sydney.edu.au Coauthors: Mary R. Myerscough

The early stages of atherosclerosis are characterized by the accumulation of lipid-laden foam cells to form a fatty lesion. The development of this lesion leads to a physiological change in the artery wall which undergoes an initial enlargement that preserves the lumenal area. We present a PDE model that involves the primary cellular species; macrophages and their lipid filled counterpart, foam cells. In the model we assume that the artery wall has a limited capacity to accommodate incoming cells and the cellular material fills a finite volume. This leads to cell sorting to form the initial lesion. We present an extension of this model which has a moving boundary and describes how the arterial wall deforms to compensate for the accumulation of cellular material.

# Modelling the Spread of a Deliberate Wolbachia Introduction

Matthew Chan University of Sydney M.Chan@maths.usyd.edu.au Coauthors: Peter S. Kim

The deliberate introduction of a specific strain of *Wolbachia* into mice populations has been recently proposed as a strategy to reduce dengue transmission. In this talk, I will discuss a reaction-diffusion approach to examine the dynamics in the spread of a *Wolbachia* infection within a population of mosquitoes in a homogeneous environment. The formulated model builds upon an earlier model by Skalski & Gilliam (2003) which incorporates a slow and fast dispersal mode. This generates a faster wavespeed than previous reaction-diffusion approaches, which have been found to produce wavespeeds that are unrealistically slow when compared with direct observations. In addition, the model incorporates cytoplasmic incompatibility between male and female mosquitoes which creates a strong Allee effect in the dynamics. In previous studies, linearised wavespeeds have been found to be inaccurate when a strong Allee effect is underpinning the dynamics. I will introduce briefly a means to approximate the wavespeed generated by the model and show that it is in close agreement with numerical simulations. These wavespeeds indicate that as the temperature decreases within the optimal temperature range for mosquito survival, the speed of a *Wolbachia* invasion increases for *Aedes aegypti* populations and decreases for *Drosophila simulans* populations. Finally, time permitting, I will discuss the population heterogeneities which give rise to leptokurtic movement distributions.

# The macroscale boundary conditions for diffusion in a material with microscale varying diffusivities Chen Chen School of Mathematical Sciences, the University of Adelaide chen.chen@adelaide.edu.au

Homogenization and other multiscale modelling techniques empower us to build efficient mathematical model for simulating materials with complicated microstructure. But the modelling rarely systematically derives boundary conditions for macroscale model. We build a smooth macroscale model for a two-layer one-dimensional diffusion system with rapidly varying diffusivity and finite scale separation. We derive macroscopic boundary conditions for this diffusion problems. The approach can be applied to a range of multiscale modelling problems including wave equations.

The role of internal pressure in microstructured optical fibre drawing Mike Chen University of Adelaide michael.chen@adelaide.edu.au Coauthors: Yvonne Stokes

The fabrication of microstructured optical fibres (MOFs) involves stretching (or drawing) a structured preform to produce long, thin fibres that feature internal air channels. A consequence of this drawing process is the deformation (and possible closure) of the internal features due to surface tension; one way to counteract this is to pressurise these internal holes. In this talk an existing model of fibre drawing will be extended to include internal pressure. This will then be used to predict the behaviour of an annular fibre for a range of drawing conditions. On the Existence of Breathers in Periodic Media: An Approach via Inverse Spectral Theory Martina Chirilus-Bruckner School of Mathematics and Statistics, The University of Sydney martina.ch.br@gmail.com Coauthors: C.E. Wayne

Breathers, i.e. time-periodic, spatially localised solutions, are known to exist for the Sine-Gordon equation, which, however, seems to be the only (constant coefficient) nonlinear wave equation to support such solutions. In this sense, breathers have been considered a rare phenomenon. Surprisingly, a nonlinear wave equation with spatially periodic step potentials has been found recently to support breathers (Blank et al. 2011) by using a combination of spatial dynamics, centre manifold reduction and bifurcation theory. Via inverse spectral theory, we aim towards characterising a larger class of potentials that allow breathers. The research is motivated by the quest of using photonic crystals as optical storage.

Modelling the evolution of uniparental inheritance of mitochondria: is mixing bad for the cell? Joshua Christie School of Biological Sciences and Centre for Mathematical Biology, University of Sydney, NSW, 2006 joshua.christie@sydney.edu.au Coauthors: Tim Schaerf, Tanya Latty, Madeleine Beekman

Mitochondria are small, membrane-bound components of cells that produce cellular energy. One of the many enduring mysteries of sexual reproduction is the post-fertilisation destruction of male-derived mitochondria, which means that only females can transmit mitochondria to offspring (i.e. transmission is uniparental). Even in single-celled species which lack defined sexes and instead have mating types, the uniparental inheritance of mitochondria (UI) is the overwhelming norm. The popular genomic conflict theory predicts that this pattern of mitochondria inheritance has evolved to mitigate conflict between the mitochondrial and nuclear genomes. If mitochondria from both parents were allowed to mix, there could be selection for fast replicating, but energetically inefficient, mitochondria. By promoting their own fitness above that of the cell, these selfish mitochondria could overwhelm the cell, inadvertently destroying it in the process. According to this theory, UI prevents the mixing of different mitochondria and safeguards the cell from a hostile takeover.

Here I present a novel hypothesis for UI by proposing that adverse effects of mitochondrial mixing, rather than the emergence of selfish mutants, best explain the evolution of UI. I will present a probability-based model that examines the spread of UI in an idealised infinite population with three mitochondrial types.

# Optimising Revenue for Concentrating Thermal Power Plants with Storage Within the Australian National Energy Market

Luigi Cirocco

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Coauthors: P. Pudney, M. Belusko, F. Bruno, J. Boland

With the proliferation of non-scheduled generation from renewable electrical energy there is an ever present need for enabling scheduled generation via the incorporation of energy storage; either as from directly coupled Thermal Energy Storage (TES) at the source or distributed Electrical Storage Systems (ESS) within the electrical network or grid.

This paper presents a brief survey of energy storage literature to establish sources for indicative storage characterisations in terms of energy capacity and cost followed by a formulation for the objective of maximising revenue of a Concentrated Solar Thermal (CST) power plant with TES within the Australian National Energy Market (NEM).

The optimal control problem formulation is presented, first as a discrete time Linear Program and then as a continuous time, state space problem resolved using the necessary conditions of Pontryagins Maximum Principle. The time intervals for control are 30 minutes corresponding to the NEM spot price for trading settlement,

being the average of the preceding 6 five minute dispatch bid prices.

The paper concludes with the selection of what we believe to be the most promising research directions to determine the optimal sizing and operation of storage within the grid for scheduled generation from renewable electrical energy sources.

#### Persistence of hydrodynamics in a two-dimensional suspension of swimming micro-organisms Richard Clarke

Department of Engineering Science, University of Auckland rj.clarke@auckland.ac.nz Coauthors: Matthew Finn

The long-ranged nature of highly-viscous flow, and its ability to couple together the dynamics of swimming micro-organisms, is well documented. When the flow regime is such that two-dimensionality is promoted, for example in thin films, we might expect these effects to become stronger still. We investigate this scenario for a suspension of simple prototype swimmers (squirmers) and show that there can indeed be significant hydrodynamic interactions in very dilute suspensions, even in the presence of decohering influences such as Brownian noise.

# Nonlinear effects on the focussing of tsunami due to underwater lenses Simon Clarke Mathematical Sciences, Monash University simon.clarke@monash.edu Coauthors: David Kinniburgh

In Berry (2007) the linear focussing of tsunami by underwater lenses was considered using diffraction theory. This proposed that under appropriate conditions underwater lenses could increase the intensity of tsunami three-fold. In this talk we consider the nonlinear propagation towards underwater lenses in the context of the variable coefficient Kadomtsev–Petviashvili (vKP) equation. Using the linear version of this equation we are able to construct equivalent asymptotic expressions for the focussing which qualitatively agree with Berry (2007), although with significantly smaller increase in the intensity. Numerical simulations of the nonlinear propagation then demonstrate that the focussing can be eliminated due to the acceleration of solitary waves over the underwater lenses. Furthermore, the vKP equation enables the downstream corrections due to finite width of the lenses to be investigated.

# Hydrodynamic Trapping Using Micro-Vortices

Jesse F. Collis The University of Melbourne, Department of Mathematics and Statistics j.collis3@student.unimelb.edu.au Coauthors: John E. Sader

A new particle trap using micro-vortices has recently been experimentally demonstrated. The micro-vortex is formed by rotating a nickel nanowire, using a weak magnetic field, to sweep through the profile of a disk. The trap operates at low Reynolds number (< 0.01) and as such is comfortably in the usual Stokes flow limit. Existing simulations using slender-body theory do not predict this behavior, as expected by the reversibility of Stokes flow. In this talk, we shall discuss the physical mechanisms behind this trap. Our results provide the ability to increase the effectiveness of these traps and develop their use in new applications.

#### Interacting large-scale atmospheric vortices

Jason Cosgrove University of Tasmania Jason.Cosgrove@utas.edu.au

The behaviour of interacting large-scale atmospheric vortices is considered. These vortices are approximately fifteen kilometres high and can have diameters of hundreds of kilometres, and so can be thought of as large flat structures. The air is weakly compressible, and the fluid motion is subject to the Coriolis pseudo-force, due to the Earth being in a non-inertial reference frame. The high or low pressure associated with the vortices is initially modelled using an exponential function. As time progresses, the vortices begin to interact. The subsequent interaction is dependent upon the separation distance and the initial sizes of the vortices. Non-linear solutions are presented using the f-plane approximation with Dirichlet and absorbing boundary conditions, and a detailed comparison is given.

Glucose Transporter Translocation - Integrating Models and Experiments Adelle C. F. Coster University of New South Wales A.Coster@unsw.edu.au

GLUT4 is an insulin-responsive glucose transporter protein and exists embedded in membrane structures of adipocytes (fat) and muscle cells. When expressed on the plasma membrane (PM) it allows the transport of glucose in and out of the cell. GLUT4 is exocytosed via small membrane vesicles, which fuse with the PM.

Live-cell imaging techniques using total internal reflection microscopy (TIRFM) mean that it is now becoming possible to view the dynamics of these processes. However, in order to systematically extract these delivery events from other background traffic, we need robust, quantitative descriptions of the dynamics.

GLUT4 may be brought to the PM via methods including the full fusion of the vesicle and the PM, and also via a process termed 'Kiss-and-Run, whereby a pore connection is made between the vesicle and PM allowing diffusion of the GLUT4 proteins across the boundary. In the latter case the vesicle does not become fully integrated and later detaches from the PM.

Models of these processes have been developed considering the diffusion of GLUT4 between the vesicle and the PM. These are then translated into the TIRFM frame of reference, enabling the automated detection and quantification of fusions in experimental data.

alpha-flips and T-points in the Lorenz system Jennifer Creaser University of Auckland j.creaser@auckland.ac.nz Coauthors: Bernd Krauskopf and Hinke Osinga

The Lorenz system is a well-known chaotic system, introduced by Edward Lorenz in 1963 as a simplified model of thermal convection. We present and characterise a phenomenon, which we call an alpha-flip where the onedimensional stable manifolds  $W^s(p^{\pm})$  of two secondary saddle equilibria  $p^{\pm}$  undergo a sudden bifurcation in their alpha-limit. Each stable manifold consists of two trajectories that converge to  $p^{\pm}$ , and their alpha-limits are the limits of these trajectories in backwards time. We follow the continuous transition of  $W^s(p^{\pm})$  which allows us to determine the moment of the the alpha-flip bifurcation and characterise it geometrically. There appear to be infinitely many alpha-flips and they are connected to infinitely many so-called T-points at which there is a heteroclinic connection from origin to  $p^{\pm}$ . These T-points are organising centers for the chaotic dynamics in the Lorenz system.

# Phase and amplitude dynamics of nonlinearly coupled oscillators.

Mr Peter Cudmore The University of Queensland peter.cudmore@uqconnect.edu.au

Large systems of coupled oscillators appear in a variety of physical systems and can produce emergent macroscopic behaviour despite small differences in the natural frequency of individual oscillators. In this talk we shall consider a coupled oscillator model originating from quantum optics whereby a large population of non-identical damped harmonic oscillators are non-linearly coupled via a population mean.

We show that this system exhibits synchronisation; in which all the oscillators rotate at the same rate. We also show how the amplitude, phase and stability characteristics of the synchronised state depend on the distribution of frequencies of the individual oscillators and on the nonlinear coupling function. We then present a geometric interpretation of these results which allows us to identify the properties of any bifurcations that may occur. In particular, this geometric description can be used to identify atypical saddle-node and pitchfork bifurcations of periodic orbits though which synchronised solutions are created.

Fractional cardiac electrophysiological models and novel techniques for their simulation Nicole Cusimano Queensland University of Technology, Brisbane nicole.cusimano@student.qut.edu.au

Successful modelling of phenomena characterized by spatial heterogeneity via fractional models in space has generated in recent years a growing interest towards fractional models and their applications.

My study currently focuses on the analysis of the effect of a fractional component in space on some cardiac electrophysiological models (commonly used in their standard formulation to model electrical pulse propagation through cardiac tissue) and the investigation of the potential of these models to capture the heterogeneous behaviour observed in some pathological cases.

Cardiac models are characterized by a coupled system of nonlinear PDEs and ODEs and already in the standard case, especially for two- and three-dimensional models, their solution is computationally expensive and usually offers several numerical challenges.

The development of a space fractional model with the introduction of a non-integer power of the second order differential operator is an additional complication and thus requires the implementation of efficient numerical techniques.

In this talk, after outlining some of the standard methods generally used to obtain a numerical solution of cardiac electrophysiological models, I consider a specific example and present its fractional modification. I then describe a novel approach for the simulation of its solution and provide some numerical results.

Free boundary model of a subglacial meltwater channel cross section Michael Dallaston University of Oxford dallaston@maths.ox.ac.uk Coauthors: Ian Hewitt

The nature of subglacial water drainage is an important component of a glacier's evolution and mass balance. One mode of drainage is via *Röthlisberger* (or "R–") channels, which are conduits maintained by the balance between the creep closure of ice and melting caused by the water flow. In glacier-scale models R–channels are generally assumed to be circular or semicircular in cross-section. In nature, however, the channels are often observed to be wide and flat.

In this talk we look at a two dimensional free boundary model of the cross section where radial symmetry is not enforced. Questions arise regarding the interfacial stability of the channel, especially as it is known that inward creep of a Newtonian fluid is unstable. The problem is also of mathematical interest as it has strong links to two dimensional Stokes and Hele–Shaw flows, for which complex variable methods have proven very successful. We discuss stability, numerical work and the occasional exact solution.

# Wavelet regularization for an inverse problem in linear viscoelasticity.

Russell Davies Cardiff University, UK DaviesR@cardiff.ac.uk Coauthors: Neil Goulding

Many inverse problems of deconvolution type have convolution kernels which are the scaling function of a continuous wavelet. In characterising viscoelastic materials such a problem occurs in which the convolution kernel is a sech function. The resulting inverse problem is exponentially ill-posed. In this talk it is shown how the famous Calderon decomposition of the identity can be used to regularize such a problem. Scale and sparsity act as simple natural regularizers in the numerical process of deconvolution. The topic of model-induced super-resolution is introduced and discussed, and numerical results presented for both synthetic and real experimental data.

# Graph Dissimilarity Measures in Biometrics

Stephen A. Davis School of Mathematical and Geospatial Sciences, RMIT University

Coauthors: Arathi Arakala and Kathy J. Horadam

We propose new dissimilarity measures to advance the characterization, classification and comparison of noisy spatial networks in general, and graph representations of ridgeline and vascular based biometrics in particular. Graph representations of the vascular pattern in human retina are constructed where nodes represent branches, cross-overs or terminations in the vasculature and two such points are connected if a vein runs uninterrupted between them. The resulting spatial, tree-like graphs are shown to provide superior matching performance compared to traditional point-pattern based approaches. We propose that this is partly due to the additional topological information stored in the graph representations, but also because matching of two graphs results in a maximum common subgraph from which multiple dissimilarity measures can be derived and used to train a machine learning algorithm. Subgraphs are then classified as arising from genuine matches or imposter matches. We hypothesized that a scale-free form of this graph-based approach would be powerful enough to provide a cheap and robust infant biometric, a notoriously challenging problem due to the growth rate of children over the first years of life. To test these ideas we collected footprints and ball prints from babies aged 0, 2 and 6 months. We will report on our progress to date on this difficult problem.

# Estimation of the approach rate from border inspection data

Geoffrey Decrouez University of Melbourne dgg@unimelb.edu.au Coauthors: Andrew Robinson, Peter Hall

Continuous sampling plans were designed to monitor and maintain the quality of a production line, and have been recently applied to monitoring the compliance of incoming goods at the border. The proportion of noncompliant units arriving at the border is known as the approach rate, and is valuable information for inspection policy, for resource allocation, and for effort spent on inspection. We develop and study the statistical properties of estimates of the approach rate, when data is collected from a continuous sampling plan. We also address the important problem of the construction of confidence bounds using the so-called split sample method. We show that the method relies on a simple theoretical property, which states that the asymptotic normal distribution approximation of the distribution a sample mean of lattice valued random variables can be improved, provided the sample is split into two subsamples, where the length of the two subsamples is carefully chosen.

# Correctly model shear-thinning fluids

Jim Denier Department of Engineering Science, The University of Auckland j.denier@auckland.ac.nz

I will correctly model the behaviour of shear-thinning fluids. Using two particular examples I will demonstrate some issues with simple power-law fluid models, often resulting in predictions of unphysical behaviour. I will resolve this problem with the development of a Carreau-type constitutive relation between the fluid viscosity and the flow shear. I will consider two particular cases; boundary-layer flow and Dean flow of a shear-thinning fluid.

# A dynamic approach to the formation of clustered ventilation defects in the lung

Graham Donovan University of Auckland g.donovan@auckland.ac.nz Coauthors: Thibaut Kritter

Clustered ventilation defects are a hallmark of asthma, typically seen via imaging studies during asthma attacks. The mechanisms underlying the formation of these clusters is of great interest in understanding asthma. Because the clusters vary from event to event, many researchers believe they occur due to dynamic, rather than structural, causes. To study the formation of these clusters, we formulate and analyze a lattice-based model of the lung, considering both the role of airway bistability and mechanisms to organize the spatial structure. We show how and why the homogeneous ventilation solution becomes unstable, and under what circumstances the resulting heterogeneous solution is a clustered solution. The size of the resulting clusters is shown to arise from structure of the eigenvalues and eigenvectors of the system, admitting not only clustered solutions but also (aphysical) checkerboard solutions. We also consider the breathing efficiency of clustered solutions in severely constricted lungs, showing that stabilizing the homogeneous solution may be advantageous in some circumstances. Extensions to hexagonal and cubic lattices are also studied.

## A Scalable Trust Region Algorithm for Nonsmooth Optimization

Andrew Eberhard *RMIT University* andy.eberhard@rmit.edu.au Coauthors: Shuai Liu and Yousong Luo

We present a special bundle trust region method for minimizing locally Lipschitz and prox-regular functions. The para convexity of such functions allows us to use the local convexification model and its convexity properties. The model is to be controlled during the iteration process such that the linearization errors are always positive. The trust region is formed by infinity norm so we have a linear subproblem in each iteration. We show that if the convexification succeeds the algorithm converges to a stationary point [and if the convexification fails then every accumulation point of the iteration sequence is stationary]. Preliminary numerical experiments on academic test problems show that the algorithm is reliable and efficient.

# A simplified approach for calculating moments of action for linear reaction-diffusion equations $\operatorname{Adam}\,\operatorname{Ellery}\,$

School of Mathematical Sciences, Science and Engineering Faculty, QUT adam.ellery@student.qut.edu.au Coauthors: Matthew J. Simpson, Scott W. McCue, Ruth E. Baker

The mean action time (MAT) is the mean of a probability density function (pdf) associated with a reactiondiffusion equation, which provides a finite estimate of the time taken for the transient solution of a reactiondiffusion equation to effectively reach steady state, also known as a critical time. Many reaction-diffusion processes have high-variance pdfs and for these problems the MAT underapproximates the critical time since it neglects to account for the spread about the mean. We can improve our estimate of the critical time by calculating the higher moments of the pdf, called the moments of action, which provide additional information regarding the spread about the mean. Existing methods for calculating the  $n^{\text{th}}$  moment of action require the solution of n nonhomogeneous boundary value problems which can be difficult and tedious to solve analytically. Here we present a simplified approach using Laplace transforms which allows us to calculate the  $n^{\text{th}}$  moment of action without solving this family of boundary value problems and also without solving for the transient solution of the underlying reaction-diffusion problem. We demonstrate the generality of our method by calculating the MAT and moments of action for several problems from the mathematical biology literature. Several of these results cannot be found using previously published techniques.

The flow of non-Newtonian fluids in a vertical oscillating cylinder Inna Elyukhina South Ural State University, Russia Inna.Elyukhina@gmail.com

We analyse the fluid flow within a right circular cylinder suspended from a thin wire and performed torsional oscillations about its vertical axis. The results are applied to oscillating-cup viscometer that is the dominant technique for liquid metals and alloys (e.g., [1]). Mathematical model of the experiment includes unsteady nonlinear differential equations of cylinder oscillations and of momentum conservation, continuity, rheological constitutive equation for fluid. It is assumed that oscillation amplitudes are small, fluid is incompressible, no wall slip (e.g., [2]). Besides numerical solutions, we also find analytical expressions for rheostable fluids under damped and forced oscillations to use them in the practice. Fluids with yield stress and non-affine deformations are studyed in detail. For the last ones, chaotic oscillations can appear. We discuss parameter identification and optimal experimental design to estimate rheological constants from measured parameters of oscillations for given viscometer geometry. The research is supported by the RFBR (N 13-08-00971). References 1. Brooks RF et al (2005) Meas Sci Technol 16:354. 2. Elyukhina I (2013) J Mater Sci 48:4387.

# Martingales with given marginals

Jie Yen Fan Monash University Jieyen.Fan@monash.edu.au Coauthors: Kais Hamza (Monash University) Fima Klebaner (Monash University)

Stochastic processes are not uniquely defined by their marginal distributions. Motivated by questions from finance, we are interested in constructing different processes (we will focus mainly on martingales) that match the marginal distributions of a given process, and we call this mimicking process. There are several approaches to construct processes with given marginal distributions. In particular, we give a construction that relies on the Markov property and the self-similarity of a given process, producing a family of martingales with the same marginal distributions. We then try to extend this construction to a broader class of processes.

# Hp-finite element methods for the radial-dependence of the linearised magnetohydrodynamic equations in spherical geometries

David Farmer School of Mathematics and Statistics, University of Sydney dfar64590gmail.com Coauthors: Dr David J. Ivers

Spherical harmonic analyses, combined with toroidal and poloidal representations of the velocity and magnetic field result in spectral forms of the linearised magnetohydrodynamic equations about a basic state. Previously developed code has focused on finite-difference or Chebyshev collocation (pseudo-spectral) methods to discretise the radial dependence of the governing equations. An alternative approach, which uses hp-finite element methods to discretise the radial dependence, is presented. The motivation for this approach is the robust exponential accuracy observed, if boundary layers are present, in idealised one-dimensional problems with the appropriate mesh-degree combination.

# Exponential Integrators and preconditioned Krylov Subspace methods for solving Fractional Diffusion Equations

Megan Farquhar *Queensland University of Technology* m.farquhar@student.qut.edu.au Coauthors: Timothy Moroney (Queensland University of Technology), Qianqian Yang (Queensland University of Technology)

In recent years there has been a surge of interest in the field of fractional partial differential equations, driven largely by the success of fractional models at describing physical phenomena that exhibit anomalous diffusion. The numerical solution of these equations can be challenging owing to the non-local dependence implied by the fractional operators.

In this work, we consider how an exponential integrator can be applied to solve a space-fractional diffusion equation defined in terms of the fractional Laplacian operator. Exponential integrators themselves have enjoyed a resurgence in the numerical community in recent years, thanks to the development of efficient methods for computing the required exponential and related matrix functions.

We demonstrate how the exponential Euler method can be used to efficiently solve the space-fractional diffusion equation which has been discretised in space with finite differences and transformed into a system of ordinary differential equations in time using the matrix transfer technique. Key aspects of our implementation include the use of contour integration methods to find products of exponentials of fractional matrix powers with vectors, Krylov subspace methods for solving shifted linear systems, outer-level preconditioning using deflation and inner-level preconditioning based on Chebyshev polynomials of the first kind applied to shifted systems. We demonstrate the effectiveness of our approach by considering a number of test problems.

Mathematical Modelling and Numerical Simulation of LiFePO<sub>4</sub> Battery Cathodes Troy Farrell QUT t.farrell@qut.edu.au Coauthors: Steven Dargaville

In this seminar we will present a two-scale, phase-field model for modern, high-rate  $LiFePO_4$  cathodes and compare this with a shrinking-core based multi-scale cathode model that we developed previously. A discussion of the many particle effects observed from the two-scale, phase-field model will also be included in this presentation.

Can Strain and Spin make a Star Bipolar? Larry Forbes School of Mathematics and Physics, University of Tasmania Larry.Forbes@utas.edu.au

Modelling the flow of fluid out from a point source is a classical problem in Applied Mathematics. However, when fluid from the source is lighter than the surrounding fluid, and there is an interface between the two fluids, the problem becomes difficult and nonlinear. Nevertheless, it can still be thought of as a canonical problem in free surface fluid mechanics. It is now known that the interface surrounding the point source is unstable at its lowest mode, so that the outflow eventually becomes a one-sided jet, regardless of initial conditions. However, in astrophysics, two-sided (bipolar) jets are often seen. Why is that? There are several possible answers, but this talk will consider straining movement of the outer fluid, and rotation of the inner fluid surrounding the point source. If the conditions are right, the one-sided jet can indeed be made bipolar. But are one-sided jets ever encountered in astrophysics?

Diffusion-driven flows in channels with arbitrarily shaped boundaries Alison French School of Mathemtical Sciences, Monash University alison.french@monash.edu

Surprisingly, when a stably stratified fluid is bounded below by an insulating inclined boundary, the lower and thus denser fluid will flow up that boundary. This phenomenon is termed diffusion-driven flow. The aim of this research is to examine the behavior of fluid, both analytically and numerically, where the inclined boundary is not straight, and to identify features of the solutions found that could be applied to other diffusion-driven flow problems. In particular, an infinitely long inclined channel with a small, isolated bump on one or both boundaries is considered. Analytic solutions are sought using asymptotic methods for the boundary layer flow, while the flow away from the boundary is calculated using two different methods which are found to produce similar results

# Canonical Duality and Triality: Unified Understanding Multi-Scale Complex Systems and NP-hard Problems.

David Gao Federation University Australia d.gao@ballarat.edu.au

Fundamental challenges in complex systems and computational science are mainly multi-scale effects of the modeling and nonconvexity of the problems involved. In static systems, the nonconvexity usually leads to multi-solutions in the related governing equations. Each of these solutions represents certain possible state of the system. How to identify the global and local stability and extremality of these critical solutions is a challenge task. It turns out that many nonconvex/discrete problems in global optimization and computational science are considered to be NP-hard. In nonlinear dynamics, the so-called chaotic behavior is due to nonconvexity of the objective functions. In complex systems, even some qualitative questions such as regularity and stability are considered as the outstanding open problems.

In this talk, the speaker will first present some fundamental principles and common misunderstandings in mathematical modeling. Based on the definitions of objectivity and isotropy in continuum physics, a potentially powerful canonical duality theory is naturally developed. Based on the traditional oriental philosophy and some basic rules in systems theory, he will show a unified framework in complex systems and the fundamental reason that leads to challenging problems in different fields, including chaotic dynamics, NP-hard problems in global optimization, and the paradox of Buridans donkey in decision sciences. By using the canonical duality theory, a unified analytical solution form can be obtained for a large class of problems in nonconvex/discrete systems, both global and local optimality conditions can be identified by a triality theory. He will show that the NP-complete quadratic integer programming problem is actually identical to a continuous unconstrained Lipschitzian global optimization problem which can be solved via deterministic methods. Based on the canonical duality theory, a new powerful primal-dual algorithm will be presented which can be used for solving a large class of challenging problems in global optimization. Applications will be illustrated by certain well-known NP-Hard problems, including maxcut problem, sensor network localization, and neural network computations etc. He will show that in complex systems, the global minimizer may not be the best solution. Finally, some common misunderstandings, open problems, and challenges on the canonical duality theory will be addressed.

The speaker hopes this talk will bring some fundamentally new insights into complex systems theory, global optimization and computational science.

References:

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[2] D.Y. Gao (2009) Canonical duality theory: Unified understanding and generalized solution for global optimization problems, Computers & Chemical Engineering, 33:19641972. doi:10.1016/j.compchemeng.2009.06.009

Higher order QMC Galerkin discretization for parametric operator equations Quoc Thong Le Gia University of New South Wales, Sydney, Australia qlegia@unsw.edu.au Coauthors: Josef Dick, Frances Y. Kuo, Dirk Nuyens, Christoph Schwab

We construct quasi-Monte Carlo methods to approximate the expected values of linear functionals of Galerkin discretizations of parametric operator equations which depend on a possibly infinite sequence of parameters. Such problems arise in the numerical solution of differential and integral equations with random field inputs. We analyze the regularity of the solutions with respect to the parameters in terms of the rate of decay of the fluctuations of the input field.

If  $p \in (0, 1]$  denotes the "summability exponent" corresponding to the fluctuations in affine-parametric families of operators, then we prove that deterministic "interlaced polynomial lattice rules" of order  $\alpha = \lfloor 1/p \rfloor + 1$  in

s dimensions with N points can be used to achieve a convergence rate of  $O(N^{-1/p})$ , with the implied constant independent of s.

# Spatial-Temporal Forecasting of Wind Farm Output

Adrian Grantham Centre for Industrial and Applied Mathematics, Barbara Hardy Institute, University of South Australia adrian.grantham@mymail.unisa.edu.au Coauthors: Manju Agrawal, Jing Huang and John Boland

In the 2012-13 financial year, the state of South Australia sourced 27% of its electricity supply from wind farms.

The work here compares five-minute and thirty-minute one-step-ahead forecasting using the coupled autoregressive and dynamical system (CARDS) and persistence forecasting, where  $x_{t+1} = x_t$ , techniques.

The key findings are firstly, clustering provides better forecasting performance compared to forecasting individual wind farm's output. Secondly, both forecasting techniques perform very well at five-minute and thirty-minute time scales. Thirdly, the CARDS forecast performs slightly better at the five-minute time scale and slightly better again at the thirty-minute time scale, particularly when the wind farm output is highly variable.

# Modelling the insulin receptor system for glucose transport Catheryn Gray

School of Mathematics and Statistics, University of New South Wales catheryn.gray@unsw.edu.au Coauthors: Adelle C F Coster (University of New South Wales, Garvan Institute of Medical Research)

Insulin is a potent peptide hormone that regulates blood glucose in the body. Adipocytes (fat cells) respond to insulin stimulation by moving glucose transporter 4 (GLUT4) to the plasma membrane, enabling the clearance of glucose from the blood. Defects in this crucial signalling pathway can give rise to insulin resistance and ultimately diabetes.

A widely cited mathematical model of insulin signalling is that of Sedaghat, Sherman and Quon (Sedaghat, 2002). Consisting of twenty deterministic ODEs, the Sedaghat model can be readily decomposed into three modular subsystems—the insulin receptor subsystem; the post receptor signalling subsystem; and the GLUT4 translocation subsystem.

Although it is the most comprehensive model of insulin signalling to date, the Sedaghat model possesses some major limitations, including the non-conservation of key components. Here we present a streamlined (reduced) model of the insulin receptor system based on the Sedaghat model. This streamlined model, with three state variables and three parameters, successfully captures the behavior of the original system, with significant gains in mathematical simplicity and computational tractability. We present the results of the analysis of the behavior of this dynamical system and compare its performance with both the Sedaghat model and several other models from the literature.

# Understanding Mild Acid Pretreatment of Sugarcane Bagasse Through Mathematical Modelling

Ava Greenwood Queensland University of Technology ava.greenwood@student.qut.edu.au Coauthors: Troy Farrell, Ian O'Hara

Sugarcane bagasse is an abundant and sustainable resource, generated as a by-product of sugarcane milling. The cellulosic material within bagasse can be broken down into glucose molecules and fermented to produce ethanol, making it a promising biofuel. Mild acid pretreatment hydolyses the hemicellulosic component of biomass, thus allowing enzymes greater access to the cellulosic substrate during saccharification.

A mathematical model describing the mild acid pretreatment of sugarcane bagasse has been developed, using a volume averaged framework. Discrete population-balance equations are used to characterise the polymer degradation kinetics, and diffusive effects account for mass transport within the bagasse material. As the fibrous material degrades over time, variations in the porosity of the material and the downstream effects on the reaction kinetics are accounted for using conservation of volume arguments.

Nonclassical symmetry solutions for reaction-diffusion equations in population genetics Dr Bronwyn Hajek University of South Australia Bronwyn.Hajek@unisa.edu.au Coauthors: Prof RJ Moitsheki, University of the Witwatersrand

Reaction-diffusion equations can be used to model problems in heat conduction, population dynamics, transmission of nerve signals and combustion theory. We use classical and nonclassical symmetry analysis to analyse a reaction-diffusion equation with spatially dependent diffusion and a cubic source term. Interesting analytical solutions can be constructed for some forms of the spatially dependent diffusion coefficient. An application to population genetics is discussed.

Analysing slow-fast systems using spectral properties Andy Hammerlindl University of Sydney andy@maths.usyd.edu.au

Multiscale or "slow-fast" dynamical systems exhibit different behaviours at different time scales. This talk will show how numerical techniques involving the Perron-Frobenius and Koopman operators and their spectra can be used to test for the presence of slow-fast behaviour and to analyse both the slow and fast dynamics in isolation.

# A Mathematical Model of Saliva Secretion

Jung Min Han University of Auckland jhan158@aucklanduni.ac.nz

Oscillations in free intracellular calcium (Ca<sup>2+</sup>) concentrations are known to act as signals in various cell types, including salivary acinar and duct cells. Specifically, a rise in free cytosolic Ca<sup>2+</sup> concentration plays an important role during early stages of saliva secretion. It is believed that these Ca<sup>2+</sup> signals are controlled by opening and closing of inositol 1,4,5-triphosphate receptors (IP<sub>3</sub>Rs) on the membrane of the endoplasmic reticulum (ER) or the sarcoplasmic reticulum (SR). We construct a mathematical model of oscillations in Ca<sup>2+</sup> concentration and the messenger, inositol (1,4,5)-triphosphate (IP<sub>3</sub>), in a duct cell line. A set of ODEs is included to portray kinetics of IP<sub>3</sub>Rs. The model captures some of experimentally observed salivary cell activities. From this model, we can postulate essential features of salivary gland function at the cellular level.

# Integrative Public Transportation Planning with LinTim Jonas Harbering University of Göttingen jo.harbering@math.uni-goettingen.de Coauthors: Anita Schöbel

Growing cities and increasing movement flows raise the need to improve Public Transportation in way that it is designed to serve the needs of passengers. Still, planning the different elements of Public Transport in one step is not (yet) possible, since the needs and conditions are too complex. The usual approach is to split up the entire problem into various subproblems, such as stop location, line planning, timetabling, vehicle scheduling, crew- and delay management, and solve them sequentially.

Having such a sequence brings up the idea of combining the different steps within a modular framework. LinTim is intended to be a toolbox, combining different approaches to each of the subproblems.

The first aim of LinTim is to solve the problem of Public Transportation Planning in a sequential process. Even more, it facilitates the investigation of an integrative Public Transportation Planning. Based on such a modular framework, "integrative" can have various mathematical shapes: Approaches can be compared, the dependencies between problems can be studied, problems can be solved iteratively or even in the same step, etc. Hence, the basic idea is to set up a framework for studying the relations that exist in the planning of Public Transportation.

# Existence of travelling waves in a model of wound healing angiogenesis Kristen Harley Queensland University of Technology ke.harley@student.qut.edu.au Coauthors: Peter van Heijster, Graeme Pettet

We discuss the existence of travelling wave solutions to a model of wound healing angiogenesis. Previous work identified a smooth travelling wave solution for specific values of the model parameters (Pettet *et at.*, IMA J. Math. App. Med., **17**, 2000). However, if these parameters were varied this solution was *cut-off* by a *wall of singularities* and hence was no longer observed. Using geometric singular perturbation theory we show that under the second parameter regime, a travelling wave solution still exists but with a shock. The existence of this shock solution relies on the appearance of a folded saddle canard point in the phase plane, which allows the solution to pass through the *wall of singularities*. We also discuss the possible existence of other travelling wave solutions for other values of the parameters where multiple canard points can coexist.

# Analysing a mathematical model of the division of labour in metabolic pathways

Emily Harvey Massey University em.harvey@gmail.com Coauthors: Tomas Gedeon, Jeffrey Heys

In naturally occurring microbial ecosystems it is often observed that the metabolic steps in the metabolism of a single substrate are divided between distinct subpopulations within the microbial community. This 'division of labour' has also been repeatedly observed to evolve from monocultures in experimental settings, and these 'consortia' are found to have increased productivity and robustness when compared to a monoculture. In this work, through analysis of a mathematical model of a prototypical system, we compare the productivity (biomass production) of a consortium where the metabolic pathways have been divided between two distinct populations of microbes, to a single super microbe which has the same metabolic capacity. We prove that if there is no change in the growth rate or yield of these pathways when the metabolism is specialised into separate microbes, the biomass (productivity) of a consortia system is always less than that of a monoculture. To explain the observed increased productivity of the consortium, we conclude that some adaptation must have occurred, and using a specific example of Escherichia coli on a glucose substrate we are able identify the metabolic changes that would have the strongest effect on increasing the productivity.

# Multitype branching processes in a random environment: Would they survive forever?

Sophie Hautphenne University of Melbourne sophiemh@unimelb.edu.au Coauthors: Guy Latouche

We consider multitype Markovian branching processes living in a random environment. It is well known that the extinction criterion of such a process is related to the conditional growth rate of the population, given the history of the environment, and that it is usually hard to evaluate. We determine upper and lower bounds using a duality approach. The upper bound appears to be often much tighter than the lower bound.

Front interactions in a three-component FitzHugh-Nagumo system Peter van Heijster QUT petrus.vanheijster@qut.edu.au Coauthors: A. Doelman, T.J. Kaper, K. Promislow

I analyze a three-component FitzHughNagumo system that was proposed previously as a phenomenological model of gas-discharge systems. This system exhibits a rich variety of dynamics of fronts, pulses, and spots. The front and pulse interactions range in type from weak, in which the localized structures interact only through their exponentially small tails, to strong interactions in which they annihilate or collide and in which all components are far from equilibrium in the domains between the localized structures. Intermediate to these two extremes sits the semi-strong interaction regime, in which the activator component of the front is near equilibrium in the intervals between adjacent fronts, but both inhibitor components are far from equilibrium there, and hence their concentration profiles drive the front evolution. In this talk, I focus on dynamicallyevolving N-front solutions in the semi-strong regime. The primary result is to use a renormalization group method to rigorously derive the system of N coupled ODEs that governs the positions of the fronts. The operators associated to the linearization about the N-front solutions have N small eigenvalues, and the N-front solutions may be decomposed into a component in the space spanned by the associated eigenfunctions and a component projected onto the complement of this space. This decomposition is carried out iteratively at a sequence of times. The former projections yield the ODEs for the front positions, while the latter projections are associated to remainders that I show stay small in a suitable norm during each iteration of the renormalization group method.

The current diabetes epidemic increases TB transmission Roslyn Hickson University of Newcastle Roslyn.Hickson@newcastle.edu.au Coauthors: K. Viney and G.N. Mercer

The evidence to support an association between tuberculosis (TB) and diabetes mellitus (DM) has been growing, highlighting the importance of the current DM epidemic as a public health problem. The risk of TB has been found to be 1.5–8 times higher in patients with DM compared to those without, for both high and low income settings. We develop a model for TB transmission with a co-morbidity of DM, exploring the effect of an increasing burden of DM on the TB dynamics.

Supercritical withdrawal through a point sink in a porous medium. Graeme Hocking Murdoch University G.Hocking@murdoch.edu.au Coauthors: Hong Zhang (Griffith University)

A spectral method is used to consider withdrawal through a point sink from a fluid with two layers of different density. The flow is supercritical, which means that both layers are flowing into the point sink. The method is easy to implement and gives the proportion of volume flowing from each layer. Importantly the lower bound on the flow is found, corresponding with the transition from single layer to two layer flow.

A mathematical model for respiratory syncytial virus transmission Alexandra B Hogan Australian National University alexandra.hogan@anu.edu.au Coauthors: Geoffry N Mercer, Kathryn Glass, Hannah C Moore

Respiratory syncytial virus (RSV) is the main cause of acute lower respiratory tract infections in infants and young children. It presents a significant health care system and economic burden, with almost all children having been infected by the time they reach two years of age.

In temperate climates, RSV dynamics are highly seasonal, with mid-winter peaks and low incidence during the summer months. However, the transmission dynamics of RSV are poorly understood and very few models for RSV that reproduce the observed patterns in the data have been published to date.

I will present an age-structured SEIRS model for RSV transmission, where the transmission rates are seasonally forced and parameters are estimated from the available literature. I will demonstrate the range of annual and biennial seasonal patterns the model can produce. I will also show how we have fit this model to a population-linked laboratory data set for the metropolitan region of Western Australia and discuss future developments for this research.

A Simple Model for Reverse Roll Coating Frank de Hoog *CSIRO* frank.dehoog@csiro.au

Reverse roll coating is used add a relatively thick protective and/or cosmetic layer on strip steel and aluminium and is widely use in industry. The key issues that need to be resolved in terms of constructing a model for model based control are:

- Calculating the total flow of paint between a rigid and an elastic roll under given operating conditions
- Calculating the paint thickness on the rolls after the paint exits the roll gap

For this application, some of the standard theory associated with elastohydrodynamic lubrication theory is not appropriate. In this presentation, models are constructed on the basis of scaling, numerical modelling, perturbation analysis and experimentation.

The Roots of Bessel Function Cross Products David Horsley School of Mathematics and Physics, University of Tasmania david.horsley@utas.edu.au Coauthors: Larry Forbes

In problems in cylindrical coordinates, there can arise a need to apply nonstandard boundary conditions to Bessel's equation. Solving these problems involves finding the roots of nonlinear combinations of Bessel Functions, known as "cross products". These functions include  $f(x) = J_n(x)Y_n(\lambda x) - J_n(\lambda x)Y_n(x)$ , and similar forms with derivatives of Bessel functions.

In this talk, we will discuss motivating problems, some properties of the roots, and a method for finding them using various asymptotic formulae.

Symplectic integrators to effective order methods Gulshad Imran The University of Auckland ggul005@aucklanduni.ac.nz Coauthors: John Butcher

A family of three stage symplectic Runge–Kutta methods are derived with effective order 4. The methods are constrained so that the coefficient matrix A has only real eigenvalues. This restriction enables transformations to be introduced into the implementation of the method so that, for large Hamiltonian problems, there is a significant gain in efficiency.

# Flow through periodic capillaries

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Coauthors: Dr Bronwyn Hajek, Prof Stan Miklavcic and Prof Lee White

Periodic tubes can act as Brownian ratchets that are potentially suitable for particle separation in fields such as biomechanics and mining processes. In some tube profiles, the fluid flow down the tube establishes a recirculation region in the expansion sections of the tube, enabling the particles to become temporarily trapped. We use the boundary element method to solve for the flow in the tube and to determine the effect of the tubes geometric parameters (shape, amplitude and wavelength of corrugation) on the flow structure. In systems where recirculation occurs, we map the size and shape of the recirculation zone.

The Incompressible Coriolis Operator and Inertial Modes in Rotating Fluids David Ivers School of Mathematics and Statistics, The University of Sydney. david.ivers@sydney.edu.au

The Coriolis acceleration of a fluid in a rotating reference frame can be treated as an operator  $\mathbf{\Omega} \times \mathbf{v} + \nabla p$ acting on incompressible flows  $\mathbf{v}$  with normal component vanishing on the boundary of the fluid volume. The eigenmodes of the Coriolis operator or "Coriolis modes", are the usual geostrophic, inertial and Rossby modes. Properties of the Coriolis operator are discussed, in particular on the class of incompressible polynomial flows. The (discrete) Coriolis modes are useful in applications if they are complete. Proof of the completeness of the Coriolis modes in a sphere is outlined. The spheroidal, ellipsoidal, periodic plane layer, periodic duct and rectangular prism are also discussed.

# Controlling viscous fingering instabilities in Hele–Shaw flows

Michael Jackson QUTm1.jackson@student.qut.edu.au Coauthors: Scott McCue

The Saffman-Taylor fingering phenomenon occurs when a less viscous fluid is injected into a Hele-Shaw cell already containing a more viscous one. Although visually appealing, the fingering is often undesirable in industrial settings. As a result, much research has recently been conducted to devise strategies to suppress these fingers. By employing one such approach, we treat the problem using a time-dependent injection rate and use linear stability analysis and conformal mapping to show that for injection rates that decrease sufficiently fast, fingering can be completely suppressed. If time permits, preliminary results for a doubly connected fluid domain will be discussed.

### Disentangling nestedness from models of complex ecosystems

Alex James University of Canterbury, NZ Alex.James@canterbury.ac.nz Coauthors: Jon Pitchford (York, UK), Mike Plank (Canterbury, NZ)

Complex networks of interactions are ubiquitous, and are particularly important in ecological communities, where large numbers of species exhibit negative (e.g. competition or predation) and positive (e.g. mutualism) interactions with one another. Recent mathematical and computational analysis has stated that nestedness in mutualistic ecological networks (i.e. the tendency for ecological specialists to interact with a subset of the species that also interact with more generalist species) increases species richness. By examining previous analytic results and applying computational approaches to 59 empirical datasets representing mutualistic plant-pollinator networks, we show that this statement is incorrect. A simpler metric the number of mutualistic partners a species has is a much better predictor of individual species survival and hence community persistence. Nestedness is, at best, a secondary covariate rather than a causative factor for biodiversity in mutualistic communities.

Instabilities of flexible channel flow Oliver Jensen University of Manchester, UK Oliver.Jensen@manchester.ac.uk Coauthors: Feng Xu, John Billingham

Flow driven through a finite-length channel, one wall of which contains a segment of membrane, can be unstable to self-excited oscillations. The character of the instability depends on whether the inlet pressure or inlet flux is prescribed. We use a spatially 1D model and 2D Navier-Stokes simulations to investigate the onset of instability under fixed inlet flux. A family of amplitude equations are derived in the neighbourhood of a degenerate bifurcation point which reveal two distinct mechanisms of oscillatory instability, one driven by static divergence and the other by resonance.

Density dependent models of microporous transport Owen Jepps Griffith University o.jepps@griffith.edu.au Coauthors: Christopher Scott

About 10 years ago, we developed an extension of Knudsen's diffusion model for zero-density-limit transport along regular micropores [Jepps, Bhatia & Searles, Phys Rev Lett 91:126102]. Extending the model to incorporate density-dependence has been a challenging source of much subsequent work [Jepps, Bhatia and Searles, J Chem Phys 120:5396; Bhatia & Nicholson Phys Rev Lett 100:236103]. Helpfully, for many practical purposes density-dependence is unnecessary because variation with density is often small, and because various practical applications (for example, in separation processes [Kumar, Jobic & Bhatia, J Phys Chem B 110:16666]) occur under conditions where the zero-density limit is an excellent approximation. Nevertheless, it is important to be able to extend the model toward density-dependence, if only to understand when the zero-density-limit is likely to be insufficient.

In this presentation I will show some recent work that we have done to address the shortfalls of our initial efforts in this area. We have focussed on a simplified model of hard spheres in pores characterised by harmonic-well-type interactions. This simplification has enabled important insights into those features of the dynamics relevant to change in transport along the pore as density increases, improve the predictive power of our model.

# The development of mathematical models to study cancer spheroid growth and drug responsiveness

Wang Jin Queensland University of Technology, Brisbane, Australia w1.jin@qut.edu.au Coauthors: Daniela Loessner, Jennifer Flegg, Helen Byrne, Dietmar Hutmacher

Ovarian cancer is considered as the leading cause of gynecological cancer deaths worldwide. The development of cancer spheroids is the major route of abdominal spread, secondary tumour growth and poor prognosis of patients. Since 3D culture models truly mimic the physiological conditions and 3D architecture of cancer spheroids, we studied the influence of therapy responsiveness on ovarian cancer spheroids in concert with in silico predictions. We treat cancer spheroid as a mixture of cells and extracellular fluid. Drug activities in the both phases are considered. An ODE system is derived from the two-phase mixture model to represent 2D monolayer cell cultures. Therefore, we study the importance of the cancer geometry by investigating the drug responsiveness in the spherical model and in the monolayer model respectively. Experimental data is fitted to the models for validation, and to determine the best fit parameter values.

The Hermite normal form for circulant and skew-circulant lattice rules Stephen Joe Department of Mathematics, University of Waikato stephenj@waikato.ac.nz

Lattice rules are used to approximate integrals defined over the s-dimensional unit cube. Corresponding to every lattice rule is the dual lattice which may be generated by an  $s \times s$  generator matrix. Previous computer searches for lattice rules of specified trigonometric degree have indicated that there is some merit in searching amongst circulant and skew-circulant lattice rules. These are rules in which these generator matrices are circulant or skew-circulant matrices .

There are many generator matrices for the same lattice, but they can be transformed using elementary row operations to a unique Hermite normal form. Here we provide a characterisation of the Hermite normal form for rank-1 circulant and skew-circulant lattice rules. Such a characterisation could be useful for restricting the number of choices in a computer search.

We also consider the converse question of whether a Hermite normal form with this characterisation yields a circulant lattice rule. Though the answer is NO in the general case, there are particular cases where the converse can be shown to be true.

# A Mathematical Analysis of the "L"-Signal Peter Johnston School of Biomolecular and Physical Sciences, Griffith University p.johnston@griffith.edu.au

The "L"–Signal has been recently introduced<sup>1</sup> as a method of determining conduction velocities in cardiac tissue. Briefly, the idea is to numerically evaluate the surface Laplacian of the electric field on the surface of the heart from time traces of potentials recorded on a regular grid of electrodes. The resulting spatially averaged time trace of the Laplacian signals shows two distinct troughs. It has been shown that the times of the maximum downslopes into these troughs enables calculation of the required conduction velocities.

In this talk I will present a mathematical analysis of this process and describe how the same results can, in principle, be obtained via a much simpler data collection system.

<sup>&</sup>lt;sup>1</sup>Dr Anders Peter Larsen, CVRTI, University of Utah, Salt Lake City, Utah, USA, (personal communication)

# Lattice-free descriptions of collective motion with crowding and adhesion

Stuart Johnston Queensland University of Technology stuart.johnston@student.qut.edu.au Coauthors: Associate Professor Matthew Simpson (QUT), Dr. Michael Plank (University of Canterbury)

Cell-to-cell adhesion is an important aspect of malignant spreading that is often observed in images from the experimental cell biology literature. Since cell-to-cell adhesion plays an important role in controlling the movement of individual malignant cells, it is likely that cell-to-cell adhesion also influences the spatial spreading of populations of such cells. Therefore, it is important for us to develop biologically realistic simulation tools that can mimic the key features of such collective spreading processes to improve our understanding of how celltocell adhesion influences the spreading of cell populations. Previous models of collective cell spreading with adhesion have used latticebased random walk frameworks which may lead to unrealistic results since the agents in the random walk simulations always move across an artificial underlying lattice structure. This is particularly problematic in high density regions where it is clear that agents in the random walk align along the underlying lattice whereas no such regular alignment is ever observed experimentally. To address these limitations we present a latticefree model of collective cell migration that explicitly incorporates crowding and adhesion. We derive a partial differential equation description of the discrete process and show that averaged simulation results compare very well with numerical solutions of the partial differential equation.

Bifurcation analysis of a model for the El Niño southern oscillation Andrew Keane The University of Auckland a.keane@auckland.ac.nz Coauthors: Claire Postlethwaite, Bernd Krauskopf

Climate models can take many different forms, all the way from very detailed highly computational models with hundreds of thousands of variables, down to more phenomenological models of only a few variables that are designed to investigate fundamental relationships in the climate system. Important ingredients in these models are the periodic forcing by the seasons, as well as global transport phenomena of quantities such as air or ocean temperature and salinity.

We consider a phenomenological model for the El Niño southern oscillation system, where the delayed effects of oceanic waves are incorporated explicitly into the model, which gives a description by a delay differential equation. We conduct a bifurcation analysis of the model in two parameters by means of dedicated continuation software. It explains some previous results and also uncovers surprisingly complicated behaviour concerning the interplay between seasonal forcing and delay-induced dynamics.

# Stochastic data assimilation methods for deriving super-resolved satellite observations of the ocean

Dr. Shane Keating University of New South Wales, Sydney, NSW 2052, Australia s.keating@unsw.edu.au Coauthors: K. Shafer Smith, New York University

Ocean flows on scales of 10-50 km play a key role in biogeochemical processes, frontal dynamics, and turbulent transport in the upper ocean. Microwave imagery of the sea-surface temperature field on these scales can be used to estimate upper ocean flow thanks to the strong correlation between surface temperature and interior potential vorticity. However, because the temperature is a smoothed version of the geostrophic streamfunction, estimates of the velocity are limited to 100-300 km in the first few hundred meters.

A method is proposed for generating superresolved sea-surface temperature images using direct low-resolution (microwave) temperature observations in combination with statistical information from high-resolution (in-frared) imagery. The method enhances the effective resolution of temperature images by exploiting spatial

aliasing and generates an estimate of small scales using standard Bayesian inference. The technique is tested in quasigeostrophic simulations driven by realistic hydrographic profiles for three contrasting regions at high, middle, and low latitudes. The resulting superresolved sea-surface temperature images are then used to estimate the upper ocean velocity field on scales of 10-50 km.

# Removal of malaria parasites by an infected host

David Khoury Centre for Vascular Research and the School of Medical Sciences, University of New South Wales david.khoury@student.unsw.edu.au Coauthors: Deborah Cromer, Kylie James, Shannon E. Best, Ashraful Haque, Miles P. Davenport

Malaria is responsible for over a million deaths each year. However, some of the most fundamental mechanisms by which the host immune system destroys parasites are still not well described. Understanding how a host removes parasitised cells is important in designing treatments and developing vaccines. Efforts to understand parasite clearance have been hampered by the fact that there does not exist an obvious way to measure parasite clearance in an infected animal. Boosting the immune response can lead to fewer circulating parasites, but it is usually unclear whether this is due to a reduction in the growth rate of the parasite or an increase in the hosts ability to remove parasites.

By combining analysis of novel experimental data with age-structured differential equation models of parasite population dynamics we are able to dissect the mechanisms of parasite clearance, and understand how clearance changes over the course of infection. This work provides a novel method for directly determining the effect of specific immune responses on parasite clearance and ultimately on the control of parasites by the host.

# Modelling evolution of post-menopausal human longevity: The Grandmother Hypothesis Peter Kim University of Sydney pkim@maths.usyd.edu.au Coauthors: John McQueen, James Coxworth, Kristen Hawkes

Human post-menopausal longevity makes us unique among primates, but how did it evolve? One explanation, the Grandmother Hypothesis, proposes that as grasslands spread in ancient Africa displacing foods ancestral youngsters could effectively exploit, older females whose fertility was declining left more descendants by subsidizing grandchildren and allowing mothers to have new babies sooner. As more robust elders could help more descendants, selection favoured increased longevity while maintaining the ancestral end of female fertility.

We develop a probabilistic agent-based model that incorporates two sexes and mating, fertility-longevity tradeoffs, and the possibility of grandmother help. Using this model, we show how the grandmother effect could have driven the evolution of human longevity. Simulations reveal two stable life-histories, one human-like and the other like our nearest cousins, the great apes. The probabilistic formulation shows how stochastic effects can slow down and prevent escape from the ancestral condition, and it allows us to investigate the effect of mutation rates on the trajectory of evolution.

## The dynamics of aircraft as ground vehicles

Bernd Krauskopf Mathematics Department, The University of Auckland b.krauskopf@auckland.ac.nz

Commercial aircraft are designed to fly, but they also need to operate efficiently and safely on the ground. Their landing gear configuration in the form of a tricycle presents specific challenges for the execution of ground manoeuvres. I will report on a collaboration with Airbus and colleagues in Bristol, where we employ dynamical systems methods to investigate stability boundaries of safe aircraft ground operation, for example, when turning off a runway at high speed.

Computational and analytical methods for gas flows generated by nanomechanical devices Daniel R. Ladiges University of Melbourne dladiges@ms.unimelb.edu.au Coauthors: John E. Sader

Recent developments in the field of micro/nano-electromechanical systems (M/NEMS) have generated increased interest in the study of small scale gas flows. At these scales the continuum assumption underpinning the Navier-Stokes equations breaks down, and the gas is rarefied. The governing equation for rarefied gas is the Boltzmann equation, which has proven difficult to solve exactly in all but the simplest cases. There are, however, numerical and approximate methods for solving the Boltzmann equation in more complex cases, which can provide useful insight into gas flows in and around M/NEMS devices.

In this talk we present a modified direct simulation Monte Carlo (DSMC) method for oscillatory gas flows that uses a hard sphere model to represent particle collisions. This advances our previous work on the relaxation approximation. In addition to this computational method, we discuss a new semi-analytical approach to approximately solve the Boltzmann equation. We will analyse several simple flow problems using both approaches, with some comparison between the two methods.

Twisted states in phase oscillator arrays Carlo Laing Massey University c.r.laing@massey.ac.nz Coauthors: Oleh Omel'chenko and Matthias Wolfrum, Weierstrass Institute, Berlin, Germany

We consider a one-dimensional array of phase oscillators with non-local coupling and a Lorenztian distribution of natural frequencies. The primary objects of interest are partially coherent states that are uniformly "twisted" in space. To analyze these we take the continuum limit, perform an Ott/Antonsen reduction, integrate over the natural frequencies and study the resulting spatio-temporal system on an unbounded domain. We show that these twisted states and their stability can be calculated explicitly. We find that stable twisted states with different wave numbers appear for increasing coupling strength in the well-known Eckhaus scenario. Superstars! Kerry Landman University of Melbourne kerryl@unimelb.edu.au Coauthors: Bevan Cheeseman, Dongcheng Zhang, Benjamin Binder, Donald Newgreen

Agent-based models and experiments suggest that stochastic competition for resources is an important concept when understanding biological processes where cell proliferation is a feature, for example in a cell invasion process. We show that a small proportion of identical initial cells accounts for a substantial proportion of the total cell population. We call these cells superstars. These results have consequences for cell fate processes.

Computing Isochrons as a Boundary Value Problem Peter Langfield The University of Auckland p.langfield@auckland.ac.nz Coauthors: Bernd Krauskopf and Hinke Osinga

We study the behaviour of vector fields that have attracting periodic orbits. A point in the basin of attraction of this periodic orbit will converge to it in phase with one particular point on the periodic orbit. This is called the asymptotic phase and all points with equal asymptotic phase form an isochron. Each isochron is invariant under the time-T flow, where T is the period of the periodic orbit. We use this invariance property to calculate isochrons accurately and reliably. This talk focusses on planar vector fields for which the isochrons are curves. We formulate the invariance of the isochrons in terms of a two-point boundary value problem which we solve using continuation. We present a new technique to find all isochrons based on the linear approximation of a single isochron.

Application of statistical inference in medical images Gobert Lee Flinders University gobert.lee@flinders.edu.au Coauthors: Mariusz Bajger

Image segmentation is the first step in a number of research fields such as image processing, computer vision and machine learning. Image segmentation can be roughly presented as the grouping of individual image pixels into (meaningful/useful) partition of regions or objects. Although a large number of algorithms and approaches have been proposed, automated image segmentation continues to be a tantalizing and challenging problem. In this paper, we look at image segmentation as an inference problem and describe the Statistical Region Merging technique (Nock et al; 2004). Applications such as CT and mammographic image segmentation will be used for illustration.

# American Exchange Options under Jump-Diffusion Dynamics

Guanghua Lian University of South Australia andy.lian@unisa.edu.au

This paper provides a pricing formula for American exchange options, where the dynamics of the underlying assets are driven by jump-diffusion processes. The American exchange option valuation problem is modelled as a free boundary problem, and is solved by decomposing the value function of an American exchange option into a European counterpart and an early exercise premium. These two components then are solved analytically. The early exercise boundary for an American exchange option proves to satisfy an algebra equation and can be solved efficiently. In this way, an asymptotic formula is derived for pricing an American exchange option. The numerical results reveal that our asymptotic pricing formula is robust and accurate.

Initial value space of integrable lattice systems and discrete Painlevé equations Sarah Lobb University of Sydney sarah.lobb@sydney.edu.au Coauthors: Nalini Joshi

In order to study solutions of discrete integrable systems it is desirable to have a geometric understanding of the space of initial values. We will provide an explicit construction of these spaces for simple examples, to illustrate the dynamics.

# Interfacial dynamics of computational models with applications to melanoma and biological tissue deformation Dylan Lusmore QUT

d.lusmore@student.qut.edu.au Coauthors: Timothy Moroney

Melanoma is the most dangerous form of skin cancer, and Australia has the highest number of new cases per capita per year worldwide. An important factor in diagnosing melanoma is the irregularity of the tumour border. Mathematical models have been proposed to describe the behaviour of a growing tumour, giving insight into what causes these irregularities to occur. This presentation will discuss one such model, detailing a travelling wave solution and the linear stability analysis of a perturbation to the moving front. A level set method will also be presented as a means of producing two-dimensional solutions numerically. A condition for the instability of the perturbed front is obtained analytically and demonstrated by the numerical results.

# Uncovering water waves with exponential asymptotics

Chris Lustri University of Sydney christopher.lustri@sydney.edu.au

Recent advances in exponential asymptotics have allowed for the study of gravity waves and capillary waves in the low-Froude and low-Bond number limits respectively. These water waves are invisible to classical asymptotic series methods, as they have amplitudes which are exponentially small in the asymptotic limit.

Here, we discuss how exponential asymptotic techniques can be used to uncover these exponentially-small waves, and the role that the Stokes Phenomenon plays in determining the free-surface behaviour. These techniques are then applied to determine the behaviour of low-Froude number gravity waves caused by steady flow over a submerged source in three dimensions. Using these methods, we recover the transverse and longitudinal waves classically associated with the Kelvin wave pattern. We then consider the equivalent unsteady problem, and show that the region of the surface containing waves may be found by considering the higher-order Stokes' Phenomenon.

# **Comparing antiviral allocation strategies** Michael Lydeamore School of Mathematical Sciences, University of Adelaide michael.lydeamore@adelaide.edu.au

During the outbreak of a novel influenza strain, antivirals are commonly used to mitigate its spread. This is typically done by allocating antivirals to all members of a household following the first confirmed case within that household. An issue with this allocation strategy is that the benefits provided by the antivirals may be wasted, as the majority of the transmission might have already occurred before antivirals are distributed to the household. Another strategy is pre-allocation, where we distribute the finite supply of antivirals to a proportion of the households as soon as the outbreak occurs. An issue with this allocation strategy is that some of the antivirals will be wasted, as some of these households may not be infected during the outbreak. I will discuss a methodological framework for comparing these strategies.

# The convergence of higher order schemes for the Projective Integration method for stiff ordinary differential equations

John Maclean University of Sydney jmac9335@uni.sydney.edu.au

Recently, numerical methods to deal with multi-scale systems have received much attention, including the projective integration method within the equation-free framework and the heterogeneous multiscale method. These powerful methods have successfully been applied to a wide range of problems, including modelling of water in nanotubes, micelle formation, chemical kinetics and climate modelling. We present a convergence proof for high-order formulations of the projective integration method for a class of deterministic multi-scale systems. We corroborate this with numerical simulations, and conclude by discussing the implications of our result when selecting a numerical method.

# Cellular Differentiation During Intramembranous Bone Formation: A Turing Pattern on a Growing Domain.

Louise Manitzky Queensland University of Technology 1.manitzky@student.qut.edu.au

Intramembranous bone formation occurs when bone forms directly without a precursor cartilage matrix. A key process required to initiate this bone formation is the differentiation of mesenchymal stem cells (MSC)s into osteoblasts, the bone forming cells. This differentiation is regulated by the presence of the proteins BMP and noggin. The relationship between BMP and noggin can be modelled as a Turing pattern. This gives rise to a chemical pre-pattern that drives the cellular differentiation to occur. Furthermore during fracture healing, intramembranous bone formation is restricted to the region between the pre-existing bone and the periosteum. In the initial stages of healing the MSCs in the periosteum proliferate driving the periosteum away from the bone surface, increasing the region in which the cellular differentiation is occurring. We incorporate this into our model by considering the effects of a growing domain on our model for cellular differentiation.

# A Geometric Approach to Eigenvalues: A New Direction in Evans Function Computations Robert Marangell University of Sydney, School of Mathematics and Statistics robert.marangell@sydney.edu.au Coauthors: K. Harley, G. J. Pettet, and M. Wechselberger

I will discuss a geometrically inspired technique for determining the stability of travelling waves. Using the example of the Fisher/KPP equation, I will produce a function (an Evans function) which acts as an instability detector. If there is time, I will discuss how to apply these ideas to travelling waves in more complicated PDEs, including Hamiltonian PDEs, coupled advection-reaction-diffusion systems, and nonlinear diffusion problems.

# Modelling the dynamics of immune escape mutation in SIV/HIV infection

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There are over 40 million people currently infected worldwide, and efforts to develop a vaccine would be improved greatly by a better understanding of how HIV survives and evolves.

One barrier to vaccination is the ability of the virus to mutate and avoid immune recognition. Immune escape occurs when the dominant strain of virus that is recognized by the immune response is replaced by an escape mutant virus, which has mutated to avoid recognition. An important question for vaccination is whether the escape mutant virus pre-exists at low levels in the host or whether it takes a long time for the mutation required to emerge.

To understand the dynamics of escape mutants and how they compete for the dominance within an infected host, we developed a stochastic-deterministic mathematical model governed by ODEs. The model was used to explore how the mechanisms of immune control and the mutation rate of the virus affect the selection of escape mutant virus. Analysis of experimental data shows a high number of escape mutants present at the time of escape. This number of escape mutants is consistent with a high mutation rate and pre-existence of mutants within the viral population, prior to immune selection.

# Case Studies of Optimal Control Problems with Delays

Helmut Maurer University of Münster, Institute of Computational and Applied Mathematics, Germany maurer@math.uni-muenster.de Coauthors: Laurenz Göllmann Department of Mechanical Engineering, Münster University of Applied Sciences, Germany

We consider optimal control problems with time delays in control and state variables. The dynamic process may be subject to control and state constraints. Necessary optimality conditions are given in the form of a Pontryagin type Minimum (Maximum) Principle. We briefly discuss discretization and nonlinear programming methods to efficiently solve delayed (retarded) optimal control problems.

The main focus in this talk is on applications of delayed control problems. The first example treats the optimal control of a two-tank continuous stirred tank reactor (CSTR) for various terminal conditions and control or state constraints. In the second example, we consider a dynamic economic model with two control variables, the delayed investment and the dividend. The goal is to optimize a utility function involving the dividend and capital stock. Optimal strategies are determined under various state constraints.

# Simulated droplet motion over virtual leaf surfaces

Lisa Mayo Queensland University of Technology lisa.mayo@student.qut.edu.au Coauthors: Scott McCue, Timothy Moroney

A thin film model is presented to simulate the motion of droplets over a virtual leaf surface, with a view to better understand the retention of agricultural sprays on plants. Three driving forces of the fluid flow are considered: gravitational force, gradients in the curvature of the leaf surface, and contact angle heterogeneities. We find that in the case of  $0.015\mu$ L droplets which are representative of a typical agricultural spray, substrate curvature plays the primary role in droplet motion, with gravitational effects only being apparent for droplets of a much larger volume. A scattering of droplets on the leaf surface is shown to lead to coalescence and hence larger droplets, while a spatially varying contact angle can cause the separation of droplets into smaller portions of fluid.

Persistence in power output of wind farm time series Lynne McArthur *RMIT University* lynne.mcarthur@rmit.edu.au

There is increasing interest in the study of wind farm power output. Many researchers have modelled these types of series using traditional methods, with varying degrees of success. In Australia, energy companies wish to predict the future 30 minutes of power that the wind farm will produce. This is a key component to financial competitiveness on the market. The objective of this study is to investigate whether these particular time series display evidence of a fractal process. This is one in which the observed dynamics of the series has long-range correlations or memory in a self-similar sense as introduced by Mandelbrot. The characteristic of these types of series is of having a long memory, where the autocovariance function has a decay rate much slower than that of standard autoregressive processes. This study is based on five minute readings of power output from wind farms. The initial findings are that there is evidence of persistence, or long memory, in the time series of South Australian windfarm power output.

# Within host influenza infection dynamics: modelling re-infection to understand the role of innate immunity

James McCaw Centre for Epidemiology and Biostatistics, Melbourne School of Population Health, The University of Melbourne jamesm@unimelb.edu.au Coauthors: Teagan Guarnaccia, Louise A Carolan, Karen L Laurie

There is suggestive epidemiological evidence that following viral clearance, rapid re-infection with influenza is possible. This observation directly challenges the prevailing immunological understanding of influenza infection, in which strain specific and sterilizing immunity prevents re-infection with the same or a related strain.

Motivated by this evidence, and using a combined experimental-theoretical approach, here we examine the kinetics of influenza infection in ferrets and characterise the opportunity for re-exposure (to homologous or heterologous strains) to re-establish infection. In doing so, we are able to model the role of both target cell depletion and the innate immunological response in clearing virus from the host.

There remain relatively few integrated experimental-theoretical studies that probe the role of the immune system in influenza viral control. Our quantitative analysis using a within-host mathematical model allows for the in vivo estimation of the role of the dynamic innate response, extending the predominant influenza within-host modelling paradigm which primarily relies on a target-cell-limited approach to data analysis.

# Mathematical models for melting nanoscaled particles need more physics to remain nice.

Scott McCue Queensland University of Technology scott.mccue@qut.edu.au Coauthors: Julian Back and Timothy Moroney

The melting temperature of a nanoscaled particle is not a constant, but instead decreases as the radius of the particle decreases. This relationship is most often modelled by a Gibbs-Thomson law, which says the decrease in melting temperature is proportional to the curvature of the solid-melt interface. Such a law must break down for sufficiently small particles, since the curvature becomes singular in the limit that the particle radius vanishes. Also, mathematical solutions to the Stefan problem with this Gibbs-Thomson law exhibit 'nasty blow-up behaviour, which is (probably) not physical. In order to make the solutions behave 'nicely, we need to add a bit more physics. I will talk about one possible way to do this.

Epidemics on Networks Karen McCulloch Massey University, Albany, New Zealand k.mcculloch@massey.ac.nz

The dynamics of an SIR (Susceptible-Infected-Recovered) epidemic process on small networks with different topological structures are investigated in order to better understand how the structure of a contact network impacts the transmission of infection throughout a population.

With a network of N nodes, the state space for an SIR model consists of  $3^N$  possible states. These states are lumped together based on symmetries of the network, in the most simple case of a complete network we can lump states together that have the same number of S, I and R nodes. For networks which are not complete we must look at the topology and the number of nodes in each infection state before determining which system states can be lumped together. Differential equations which describe the transition of the network between the lumped states are derived. The individual final size probabilities are found analytically and a final size distribution produced for each of the small networks. The final size is the total number of individuals that were infected at some point during the epidemic. We also numerically solve the set of differential equations for each network and confirm our analytic results.

#### Instability in sloping layered warm-water aquifers

Robert McKibbin Institute of Natural and Mathematical Sciences, Massey University, Albany campus, Auckland, New Zealand R.McKibbin@massey.ac.nz

Thermally-driven convection in sloping permeable layers is a phenomenon that may appear in many industrial and geophysical systems. It is worth noting that most groundwater aquifers have some degree of tilt caused by crustal movement after a sequence of alluvial layers is deposited. The aim here is to take advantage of the natural bedded structure of such geological systems in the formulation of a mathematical model.

Earlier, some questions were posed about fluid and heat flows in a geologically-stratified groundwater aquifer with a small slope, when it is subjected to a perpendicular temperature gradient. The strength of shear flows in the direction of the maximum slope and the associated advected heat flux were calculated. This talk continues the discussion, in particular about the stability of such flows and the shape and amplitude of convective rolls that may form when the critical Rayleigh number is exceeded. The models discussed here are for buoyancydriven fluid flow in long, sloping warm-water aquifers containing either a homogeneous but anisotropic medium, or structures that are smoothly- or discretely-layered.

### On or Off: Simplifying Ion Channel Modeling

Dr. Shawn Means Auckland Bioengineering Institute s.means@auckland.ac.nz

Biological cells deploy ion channel proteins responding to varying concentrations of ligands (such as calcium or neurotransmitters) or membrane voltages for myriad signaling mechanisms. Markov chains are traditionally used to model their behaviour where individual nodes in the chain represent configurations of either ion-permeable (open) or non-permeable (closed) states; notably, some published models include relatively large numbers of open and closed state representations. Simulation methods include the classic Monte Carlo scheme steadily marching through time, or the more efficient Gillespie scheme (1977) that exploits probability distributions of state changes and leaps across time to the precise moment of a transition. However, the only states relevant to a cellular signaling mechanism is whether a channel is open or closed and not necessarily all the possible states included in a Markov chain. We thus present preliminary investigations of representing a channel with its frequency of opening and closing instead of modeling all individual states. A Gillespie-inspired method is used where power spectral densities are computed and provide channel frequency distributions. Our aim is simplifying simulations of ion channel behaviour by excluding non-relevant states as coupled within an overall cell model that may represent mechanisms ranging from neurotransmitter release to muscle contraction.

### Weakly Dispersive Internal Waves

Peter D. Miller Dept. of Mathematics, University of Michigan millerpd@umich.edu Coauthors: Z. Xu

Asymptotic models for internal wave motion in 1+1dimensions include nonlocal linear dispersion terms arising from theelimination of potential flow on one side of the interface via a Dirichlet-Neumann map. Such models include the intermediate long waveequation in the case of finite depth and the Benjamin-Ono equation in the case of infinite depth (of the lower fluid layer). In somesituations it is physically reasonable to assume that the dispersiveeffects are formally small compared with nonlinear effects thateventually lead to wave breaking, and then it is interesting to study the effect that weak dispersion has as a regularizing effect on the breaking waves. This problem has been studied for many years in the context of the Korteweg-de Vries equation, with key ideas going backto Whitham, Gurevich-Pitaevskii, and Lax-Levermore, and with more modern developments such as the results of Claeys and Grava arising from the Deift-Zhou steepest descent method for Riemann-Hilbertproblems. In this talk, I will describe some of our attempts to study the corresponding problem in the context of the Benjamin-Ono equation. In particular, we will present a simple and intuitive weakconvergence result (joint work with Z. Xu) that is a consequence of anew analogue of the variational method of Lax and Levermore but thattakes as inspiration also the moment expansion method of Wigner inrandom matrix theory.

### **Despeciation in Phylogenetics**

Jonathan Mitchell School of Mathematics & Physics, University of Tasmania jm06@utas.edu.au Coauthors: Barbara Holland and Jeremy Sumner

Phylogenetics involves reconstructing the evolutionary history of species. By comparing DNA sequences for each species, we can create a phylogenetic tree or network that describes the evolutionary history of those species. We assume that the nucleotide bases, A, C, G and T, mutate between one another at constant rates under a Markov model of evolution. This allows us to construct a probability distribution, the phylogenetic tensor, for combinations of nucleotide bases across species. A question that has recently arisen in phylogenetics is whether "despeciation," the process whereby species become more similar to each other, e.g. via hybridisation, can be detected. Intuitively, it is not clear whether speciation, followed by a period of independent evolution at a slower rate. To describe despeciation, I will examine a particular Markov model of evolution under the network scenario [Sumner et al., 2012]. I will show that by employing methods from algebraic geometry, namely Groebner bases, we can solve the polynomial equations which define the phylogenetic tensor and address the question of whether despeciation can be detected on small phylogenetic networks.

You're As Cold As Ice: A tale of multiple timescales. Mr John Mitry The University of Sydney john.mitry@gmail.com Coauthors: Martin Wechselberger

We present an analysis of the Huber-Braun Cold Receptor Model (HB CRM; Finke, et al. 2010). The HB CRM is a 4-dimensional modified Hodgkin-Huxley system which models mammalian temperature sensitivity. The model displays a diverse range of spiking behaviour under variation of temperature. Using geometric singular perturbation theory (GSPT) we aim to elucidate the mechanisms by which this model can exhibit tonic spiking, chaotic bursting, non-chaotic bursting and sub-threshold oscillations as temperature increases. While the HB CRM is a singularly perturbed problem - making possible an analysis via GSPT - we find that the timescale structure is not preserved under variation of temperature. This shifting of timescales is compounded by, and possibly the reason for, ranges of temperature values for which a 3 timescale analysis is required in order to best illustrate the dynamics observed.

# An energy-preserving finite-difference scheme for the modified Hunter-Saxton equation Yuto Miyatake The University of Tokyo yuto\_miyatake@mist.i.u-tokyo.ac.jp Coauthors: David Cohen, Daisuke Furihata, Takayasu Matsuo

In this talk, we propose an energy-preserving finite-difference scheme for the modified Hunter–Saxton equation. This equation is a model for short capillary waves propagating under the action of gravity, and has bi-Hamiltonian and other rich mathematical structures. An interesting feature of this equation is that it admits travelling waves, and then our main goal is to simulate travelling waves by using an energy-preserving scheme. However, this task is not straightforward mainly because solutions of the equation are not unique. For this problem, we show that we can extract travelling wave solutions by using a pseudo-inverse operator.

Modelling the Distributions of Competing Species along Environmental Gradients Mohd Hafiz Mohd University of Canterbury hafiz.mohd@pg.canterbury.ac.nz Coauthors: Rua Murray (University of Canterbury) Michael Plank (University of Canterbury) William Godsoe (Lincoln University)

Elucidating the factors that shape the distributions of species has long been a central focus in ecology. There are several important factors that can influence species distributions, namely competitive interactions and abiotic environment. Competitive interactions have the capacity to generate range limits (i.e. the boundaries of locations in which a species is found). The change in environment can be associated with numerous abiotic factors such as temperature, moisture and elevational height. We study a simple model of biotic interactions consisting of interspecific competition and environmental suitability (carrying capacity) terms. We show that the possible outcomes of species interactions depend on the strength of interspecific competition and the rate of change of parameters in space.

# Preconditioning techniques for efficient method of lines solutions of space-fractional partial differential equations

Tim Moroney Queensland University of Technology t.moroney@qut.edu.au Coauthors: Qianqian Yang, Alex Simmons

A common and convenient means for solving parabolic PDEs is spatial discretisation followed by temporal integration using a variable stepsize ODE solver – the method of lines. Implicit methods are preferred because they allow larger steps to be taken than their explicit counterparts. Jacobian-free Newton-Krylov methods are favoured for solving the resulting nonlinear systems, but effective preconditioning is crucial to obtaining good performance. In the case of space-fractional PDEs, the construction and application of preconditioners requires careful treatment in order to avoid costly formation and factorisation of dense Jacobian matrices. We examine several examples of PDEs built with fractional differential operators and show how to construct effective preconditioners by appealing to other, related fractional operators. The preconditioners are easy to embed within existing ODE frameworks such as CVODE, leading to high performance solvers.

# Mechanism Design and the $<\!N,\!M\!>$ Trade Problem Ellen Muir

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We examine the Walrasian (or efficient) outcome of the  $\langle N,M \rangle$  setup, a trade model for a market with N buyers and M sellers. The problem is motivated in the context of mechanism design by introducing basic mechanism design theory related to the  $\langle N,1 \rangle$  setup (an auction with N buyers). The  $\langle N,M \rangle$  setup may be analysed by reformulating the problem in terms of two independent sets of order statistics. This enables the distribution of the Walrasian quantity traded to be computed. Emphasis is placed on the combinatorial nature of the problem under certain conditions. An exact expression for Walrasian welfare can be determined by exploiting order statistic properties related to Markov processes. Finally, Walrasian welfare in large markets is estimated by utilising results from large sample theory to complete an asymptotic analysis.

# Variability in long-term HBV dynamics under antiviral therapy

Professor John M Murray School of Mathematics and Statistics, UNSW Australia J.Murray@unsw.edu.au Coauthors: O. Stancevic, M. Lutgehetmann, J. Petersen, M. Dandri

Our understanding of the effects of antiviral therapy for individuals infected with hepatitis B virus (HBV) has been inferred from the dynamics of HBV DNA levels in serum. Studies of HBV DNA levels with a variety of drugs when followed over periods of 4 to 12 weeks show multiphasic decay that can be quite complex, but are generally biphasic. However analyses over longer-time frames have been missing. Furthermore these studies do not include any data on HBV levels in the liver but infer them from serum HBV DNA levels. In this study we analyse serum HBV DNA dynamics from chronically infected individuals undergoing 1 year of combination therapy with pegylated interferon alpha and adefovir dipivoxil (ADV), followed by 2 years of ADV monotherapy. We find that the complex patterns of decay observed over shorter time frames, are carried over to this longer period of investigation and are typified by large oscillations in serum viral levels that are not reflected in the sustained decreases in the liver. Mathematical modelling of these dynamics produce simulations consistent with these data by assuming outgrowth of viral clones during therapy that are initially unrecognised by the humoral response.

Computation of ergodic objects in dynamical systems Rua Murray University of Canterbury rua.murray@canterbury.ac.nz

The ergodic theory of dynamical systems allows, in principle, for making precise statements about the long-time average behaviour of functions on phase space. Practical application of these ideas requires knowledge of the invariant probability measures supported by a dynamical system, as well as insight into which measures are the important ones. This talk will review projection and variational schemes (Ulam's method and MAXENT, respectively) for approximating invariant measures numerically, as well as surveying what is known about their convergence, reliability, usefulness and potential.

Numerical study of SARS SEIJTR epidemic model with the inclusion of diffusion in the system Afia Naheed Swinburne University of Technology

anaheed@swin.edu.au Coauthors: Manmohan Singh, David Lucy

In recent years, there is an increasing trend of utilizing mathematical models for the prediction and insight of infectious diseases. These models are considered as a conceptual tool to explain the behavior of a disease at different stages, which allows us to understand the spread of infection along with the impact of various factors on disease dynamics. Such models help specially in cases when much biological information and treatment is not available. Our work is based on a numerical study of Sever Acute Respiratory Syndrome, *SARS*, using an open population model consisting of six compartments *SEIJTR* (susceptible, exposed, infected, diagnosed, treated and recovered). Different initial conditions on population distributions are used to generate the numerical simulations for the disease. Operator splitting technique is used to obtain the numerical solutions. Stability of disease free and endemic equilibrium of the system is established. In order to have a look on the possibilities to control the transmission of disease, five different cases have been considered. It is observed that if using some intervention or non intervention strategies with a decrease of transmission rate, then infection spreads slowly in the population. Also with decrease in rate of progression from infective to diagnosed, there happens a reduction in the recovery. An increase in transmission rate leads to an increase in infected population quite quickly. An increase in diagnosis rate also leads to an increase in recovery.

The lattice equations arising from the  $\tau$  functions of q-P $(A_5^{(1)})$ Nobutaka Nakazono The University of Sydney nobua.n1222@gmail.com

Relations between ABS equations and discrete Painlevé equations are well known e.g. the periodic reduction. In this talk I will show that the lattice equations called LMKdV (or  $H3_m^{\delta=0}$  in the ABS list) is derived from the  $\tau$  functions of q-Painlevé equations of surface of type  $A_5^{(1)}$ . This work supported by the Australian Research Council grant DP130100967.

Three Time Scale Phenomena Pingyu Nan University of Auckland pnan011@aucklanduni.ac.nz

We are interested in the dynamics of a GnRH neuron model, which was constructed to understand electrical activity and calcium oscillations in GnRH neurons. As in many other mathematical models of physiological systems, the GnRH neuron model has multiple timescales. Typical solutions appear to involve at least three different timescales. Methods for the analysis of models with two timescales are now well established, but little is known about the case of three or more timescales. This fact motivates us to make progress on understanding systems with three timescales. In particular, we would like to identify intrinsically three timescale phenomena.

In this talk, we will introduce multiple timescale systems in the context of the Morris-Lecar equations, and show the progress that has been made on understanding three time scale oscillations in a coupled Morris-Lecar model.

### The effect of Wolbachia on dengue transmission

Meksianis Ndii The University of Newcastle, Australia meksianis.ndii@uon.edu.au Coauthors: Roslyn Hickson, David Allingham, Geoffry Mercer, Irene Hudson

Using the bacterium Wolbachia is an innovative new strategy to break the cycle of dengue transmission. There are two main mechanisms by which Wolbachia could achieve this; by reducing the level of dengue viruses in mosquitoes, and by shortening the mosquito lifespan. Although Wolbachia shortens the mosquito lifespan, it also gives a breeding advantage which results in complex population dynamics.

This study focuses on the development of a mathematical model to quantify the effect on human dengue cases of introducing Wolbachia into the mosquito population. The model consists of a compartment-based system of first order differential equations with seasonal forcing in the mosquito population through the adult mosquito death rate. The analysis focuses on a single dengue outbreak typical of a region with a strong seasonally varying mosquito population. We found that a significant reduction in human dengue cases can be obtained provided Wolbachia carrying mosquitoes persist when competing with mosquitoes without Wolbachia. When all other effects of Wolbachia on the mosquito physiology are ignored, Cytoplasmic Incompatibility results in a reduction in the number of human dengue cases.

Sludge Disintegration in the Activated Sludge Process Mark Nelson University of Wollongong mnelson@uow.edu.au Coauthors: A.O.M. Alharbi, A. Worthy and H.S Sidhu

One drawback associated with the activated sludge process is the production of 'sludge'. The expense for treating excess sludge can account for 50-60% of the total operating costs in a wastewater treatment plant. Traditional methods for disposing of excess sludge, which include incineration, the use of landfill sites and dumping at sea, are becoming increasingly regulated due to environmental concerns about the presence of potentially toxic elements in the sewage sludge. Furthermore, the costs of using landfill sites, particularly in urban areas, have increased sharply. Thus there is a growing interest in methods that reduce the volume and mass of excess sludge produced as part of biological wastewater treatment processes.

We use a standard model for the activated sludge process in which the influent contains a mixture of soluble and biodegradable particulate substrate. Within the bioreactor the biodegradable particulate substrate is hydrolysed to form soluble substrate. In turn the soluble organics are used for energy and growth by the biomass. Dead biomass flows into two compartments: soluble substrate and an inert fraction.

We investigate the use of a sludge disintegator unit to control the amount of sludge, the sum of insoluble substrate, inside the reactor.

Pattern formation on the cell cortex Zoltan Neufeld University of Queensland z.neufeld@uq.edu.au Coauthors: Alpha Yap, Selwin Wu, Guillermo Gomez

The cell cortex is a biologically active medium composed of actin filaments and myosin molecules that generate contractile forces. Similar actin-myosin systems also play important role in muscle contraction or in the formation of protrusions of crawling cells. We investigate pattern formation in a partial differential equation based continuum model of the acto-myosin system particularly focussing on the role of the turnover of actin polymers in generating a dynamically changing network of actin cables. These dynamic patterns appear to be similar to

oscillations observed in recent experiments using epithelial cell monolayers, and may have functional roles in multicellular processes such as wound closure, collective migration or extrusion of cancer cells.

A multidimensional Fokker-Planck equation description of energy conversion in molecular motors Phuong T. T. Nguyen SCION, 49 Sala Street, Rotorua pngu023@aucklanduni.ac.nz Coauthors: Katherine J. Challis, Michael W. Jack

Energy use in biological systems is facilitated by specialised motor proteins (ion pumps, rotary motors, and cytoskeletal motors) that are extraordinarily efficient. These motors operate far from equilibrium in an environment dominated by thermal fluctuations, where directed motion is as difficult as walking in a hurricane and swimming in molasses (Astumian, 2007). The mechanism by which motor proteins efficiently use chemical energy intrigues scientists from many research areas spanning both the physical and life sciences. We describe energy conversion in a molecular motor in terms of Brownian motion on a multidimensional tilted periodic potential. The Fokker-Planck equation for this system is not analytically solvable in general. However, in the case of deep potential well, the continuous multidimensional equation transforms to a discrete master equation that is analytically solvable. Our current focus is deriving analytic solutions to this master equation for a range of different cases and interpreting these solutions to test the theory against the results of experimental studies on ATP-synthase, a crucial molecular motor in living systems. We present an overview of our theoretical formalism and preliminary results of our ATP-synthase study.

QMC methods for PDEs with random coefficients James Nichols University of New South Wales james.ashton.nichols@gmail.com Coauthors: Frances Kuo, Ian Sloan, Rob Scheichl, Christoph Schwab, Ivan Graham

We present recent results in the theory of applying QMC rules to integrating PDEs with random coefficients, as for example arises in the Darcy flow problem in porous media. Proving good convergence of rank-1 lattice rules requires new results for generalised weighted Hilbert spaces. We present those results, and discuss decisions motivated by the challenging numerics of this problem.

### Numerical Random Matrix Theory

Sheehan Olver School of Mathematics and Statistics, The University of Sydney sheehan.olver@gmail.com

Random matrix theory has undergone significant theoretical progress in the last two decades, including proofs on universal behaviour of eigenvalues as the matrix dimension becomes large, and a deep connection between algebraic manipulations of random matrices and free probability theory. Underlying many of the analytical advances are tools from complex analysis. By developing numerical versions of these tools, it is now possible to calculate random matrix statistics to high accuracy, leading to new conjectures on the behaviour of random matrices. We overview recent advances in this direction.

### Computing failure boundaries by continuation of a two-point boundary value problem

Hinke Osinga *The University of Auckland* h.m.osinga@auckland.ac.nz Coauthors: Anna Spyker, The University of Auckland

We propose a novel approach to investigating the parameter dependence of system behaviour for models that are subject to an external forcing. As a particular example we consider the analytical model of a tied rocking block on an elastic foundation, which exhibits dynamics equivalent to that of a planar, post-tensioned frame on a shake table; here, we consider a periodic external force, but our goal is to predict behaviour of models subject to an aperiodic external force (an earthquake). Initial conditions, given by the angle and angular velocity of the rocking block, are said to fail if the trajectory through this point moves past a given maximum angle. A standard approach would be to run simulations over a grid of initial conditions. We are interested in the more efficient method of computing the failure boundary directly. To this end, we set up a two-point boundary value problem for which the solutions graze this maximum-angle boundary. We find that the failure boundary is only piecewise smooth and are able to describe its properties in detail.

## Recent advances in multi objective mixed/integer/linear programming Dr. Melih Ozlen *RMIT University* melih.ozlen@rmit.edu.au

It has been a very exciting couple of years for the field of multi objective mixed/integer/linear programming with a renewed interest in the area: New types of solutions, unsupported, non-extreme supported, etc., which were omitted by the previous generation of algorithms, were identified. New algorithms were developed to generate efficient or nondominated sets. New methods to optimise functions over the nondominated or efficient sets were proposed. Problems with more than three objectives were solved for reasonably large instances. Some old algorithms were revisited to improve their performance significantly or to fix their shortcomings. Algorithms taking advantage of highly parallel computing environments were developed. Bounds were proposed on the number of subproblems that need to be solved to generate nondominated sets. Open-source implementations of some multi objective programming algorithms were released. In this talk, we will discuss these exciting years, comment on the current state of research in multi objective mixed/integer/linear programming, and will identify some future research directions.

The flow of a rotating fluid at low Rossby number Michael A Page School of Mathemtical Sciences, Monash University Michael.Page@monash.edu

I return here to some work that I mostly completed many years ago, to tie up some loose ends. It concerns fluid motion in a rapidly rotating viscous fluid at low Rossby numbers, where dominantly depth-independent flow can be described using potential theory for the irrotational flow on a broad scale in combination with a form of nonlinear boundary-layer theory close to any solid boundary. The flow structure is like that used for the forms of two-dimensional high Reynolds number flow that are often seen in an undergraduate fluid dynamics subject, but with an additional decelerating force arising from the rotation. As the scaled Rossby number is increased, the boundary-layer flow near an isolated obstacle changes from being fully attached and governed by a linear equation (typically with an exact solution) to becoming separated and then reattaching downstream. In different situation, a thin isolated jet of high speed fluid is found to detrain all of its momentum after a finite distance, unlike the classical 'Bickley jet' which slows down gradually downstream. These simple steady flows are pretty well understood, but what happens to more complicated flows, or when unsteadiness arises?

### Filtering the maximum likelihood for multiscale problems

Andrew Papanicolaou University of Sydney alpapani@maths.usyd.edu.au Coauthors: K. Spiliopoulos

Filtering and parameter estimation under partial information for multiscale diffusion problems is studied in this paper. The nonlinear filter converges in the mean-square sense to a filter of reduced dimension. Using this result, we establish that the conditional (on the observations) log-likelihood process has a correction term given by a type of central limit theorem. We prove that an appropriate normalization of the log-likelihood minus a log-likelihood of reduced dimension converges in distribution to a normal distribution. To achieve this we assume that the operator of the (hidden) fast process has a discrete spectrum and an orthonormal basis of eigenfunctions. Based on these results, we then propose to estimate the unknown parameters of the model based on the reduced log-likelihood, which is an easier function to optimize because it is of reduced dimension. We also establish consistency and asymptotic normality of the maximum likelihood estimator. Simulation results illustrate our theoretical findings.

Travelling Waves and Oscillations in Combustion Reactions Rhys A. Paul School of Mathematics and Physics, University of Tasmania rpaul@utas.edu.au Coauthors: Larry Forbes

It is well known that the Salnikov reaction model can present varied and complex behaviours. In the case of well mixed gases, large amplitude relaxation cycle solutions have been studied thoroughly and were experimentally verified by Coppersthwaite et al. Recently Forbes used asymptotic methods to demonstrate that when the gases are also subject to the governing equations of fluid dynamics (Eulers momentum equation and conservation of mass) small amplitude perturbations to the steady state may self-organise, forming travelling waves with a sech-squared profile. We present an extension of this method to gases which are viscous and subject to gaseous diffusion, and show that the sech-squared travelling wave remains. We also present a numerical scheme which we use to track accurately the evolution of the system and to verify the asymptotic solution. This scheme is then used to investigate parameter regions not covered by the asymptotic analysis and pulsatile travelling wave solutions are found, in which multiple travelling waves with periodically varying amplitude and wave speed branch out from the initial perturbation.

Taking interacting motile agents beyond monomers and nearest neighbour steps Catherine Penington University of Melbourne c.penington@student.unimelb.edu.au Coauthors: Barry D. Hughes and Kerry A. Landman

Most agent-based models on lattices use monomer agents moving only to their nearest-neighbour sites. Discarding this restriction, we consider agents with length L attempting movements over a distance d and obeying a simple exclusion rule. Nave mean-field approximations that work well for monomer agents are increasingly inaccurate for longer agents, so we present a new and more accurate alternative. In the continuum limit, nonlinear diffusion equations that describe the average agent occupancy are obtained, and solutions of these compare very well with average discrete simulation data. This framework allows us to approach a lattice-free result using all the advantages of lattice methods. Since different cell types have different shapes and speeds of movement, this work offers insight into population-level behaviour of collective cellular motion.

# Jacobian-free Newton-Krylov methods with GPU acceleration for computing nonlinear ship wave patterns.

Ravindra Pethiyagoda Queensland University of Technology ravindra.pethiyagoda@student.qut.edu.au Coauthors: Scott W. McCue (Queensland University of Technology) Timothy J. Moroney (Queensland University of Technology) Julian M. Back (Queensland University of Technology)

The talk is concerned with computing steady three-dimensional ship wave patterns that form when a uniform stream flows past a point source. Flows of this sort are of great interest to the design of ship hulls or submarines, for example. By applying a boundary integral approach, the nonlinear free surface problem is reduced to an integral equation in two spatial dimensions, which is solve together with a Bernoulli-type boundary condition on the unknown fluid surface. The problem is discretised into a system of non-linear algebraic equations which is solved using Newton's method and variants. We concentrate on applying a preconditioned Jacobian-free Newton-Krylov (JFNK) method with function evaluation using a GPU, and show that results can be computed much more efficiently and accurately than those presented in contemporary literature.

Vascular angiogenic fronts - modelling insights into structure and function Graeme Pettet Queensland University of Technology g.pettet@qut.edu.au

Vascular or capillary fronts are a feature of both tumour-induced and wound-healing related angiogenesis. Such fronts exhibit a microscopic architecture that defies easy experimental analysis and hence rational theoretical explanation. Here are presented continuum modelling and discrete simulation results that indicate that the behaviour of the travelling wavefront of endothelial cells, as well as the structure within the vascular front, are attributable to cytokines induced proliferation and migration. Analysis of the underlying mathematical models leads to conclusions regarding the structural elements such as the formation of a brush-border to the wavefront in tumour-induced angiogenesis. Implications for hyper-proliferative diseases are also considered.

Langmuir circulation in sheared shallow waters W.R.C. Phillips Swinburne University of Technology wphillips@swin.edu.au

The instability of shallow water waves on a sheared substrate to stadium sized rolls known as Langmuir circulation is considered. In such instances the shear can significantly affect wave-wave interactions measured through an averaged nonlinearity known as drift to realize profiles markedly different from the simple Stokes drift. Since drift and shear are instrumental in the instability to Langmuir circulation, of key interest is how that variation in turn affects onset to Langmuir circulation. Also of interest is the effect to onset of various boundary conditions, viz Neumann and Cauchy. Two typical flow fields are considered, namely shear driven and current driven flow. Relative to the reference case, shear driven flow is found to be destabilizing and current driven stabilizing to Langmuir circulation. In current driven flow it is further found that multiple layers, as opposed to a single layer, of Langmuir circulation can form. Moreover the layers can extend into a region of flow beyond that in which the instability applies. Finally, while Neumann-Neumann are known to ensure the least stable spanwise wavenumber is zero and Cauchy-Neumann boundary conditions non zero, we find the latter further act to realize the long observed but unexplained large aspect-ratio shallow water Langmuir circulation.

Supported by grants from the NSF and ARC

### Modelling risk in electricity markets

Andy Philpott Engineering Science a.philpott@auckland.ac.nz Coauthors: Corey Kok

Electricity markets are supposed to provide incentives for agents to act so as to maximize the sum of long-term welfare for all participants. The future is uncertain, and so welfare must be measured in terms of some risk measure (such as mathematical expectation). When the agents are risk-averse and markets are incomplete, markets can fail to deliver this outcome, even when they are perfectly competitive. We describe some simple complementarity models that illustrate these effects, and give some examples to show how these can be used to model behaviour in the New Zealand electricity market.

# Subharmonic Pattern formation and drifting in a quasi one dimensional periodically driven shallow granular bed

Ignacio Ortega Piwonka University of New South Wales ignacio.ortega.piwonka@gmail.com Coauthors: Marcel Clerc, Claudio Falcn, Jeremas Garay

We present an experimental and theoretical study of the pattern formation process of subharmonic waves in a quasi-one-dimensional shallow granular bed fluidized by means of a time-periodic air flow. Measurements of the amplitude of the critical mode close to the transition are in good agreement with those inferred from a universal stochastic amplitude equation. This allows us to determine both the bifurcation point of the deterministic system and the corresponding noise intensity. We also show that the probability density distribution is well described by a generalized Rayleigh distribution, which is the stationary solution of the corresponding Fokker-Planck equation of the universal stochastic amplitude equation that describes our system.

Depending on the tilt angle of the granular layer, the patterns may stand still or exhibit drift motion. This pinning-depinning transition is mediated by the competition of the inherent periodicity of the subharmonic pattern, the asymmetry of the system, and the finite size of the cell. Measurements of the mean phase velocity of the subharmonic pattern are in good agreement with those inferred from an amplitude equation, which takes into account asymmetry and finite-size effects of the system, emphasizing the main ingredients and mechanism of the transition.

Smaller fish to fry? Michael Plank University of Canterbury michael.plank@canterbury.ac.nz

Most commercial fisheries operate by selectively targeting big fish and protecting small fish from capture. There are disadvantages with this approach: it disrupts the natural size structure of the population; it can destabilize the population, resulting in unpredictable fluctuations; it is causing fish to evolve towards smaller sizes. I will talk about an alternative approach called balanced harvesting. The idea is to catch a range of fish species and sizes in proportion to their natural abundance and productivity. In practice, this means catching lots of small fish and fewer big fish. I will use a size-structured model to compare the outcomes from traditional fishing methods and from balanced harvesting. The models predict that balanced harvesting can give a higher yield than current fishing practice, with less disruption to the population. I will also show the results of a multi-species model and show that some counterintuitive things can happen as a result of the fact that fish can grow by around 8 orders of magnitude over their lifetimes.

### A mathematical model of pigeon navigation

Claire Postlethwaite University of Auckland c.postlethwaite@auckland.ac.nz Coauthors: Michael Walker

How animals find their way whilst navigating for homing or migration has intrigued scientists for many years. Due to their ease of handling and willingness to home, the homing pigeon is an ideal candidate for study, and the recent advent of miniaturised GPS devices has made available large quantaties of high quality tracking data. We present a new mathematical model for animal navigation, which explains some interesting features of GPS-captured pigeon homing trajectories.

Rectification of self-propelled particles by symmetric barriers Andrey Pototsky Swinburne University of Technology, Melbourne Australia apototskyy@swin.edu.au Coauthors: A. M. Hahn and H. Stark

The motion of self-propelled particles can be rectified by asymmetric or ratchet-like periodic patterns in space. Here we show that a non-zero average drift can already be induced in a periodic potential with symmetric barriers when the self-propulsion velocity is also symmetric and periodically modulated but phase-shifted against the potential. In the adiabatic limit of slow rotational diffusion we determine the mean drift analytically and discuss the influence of temperature. In the presence of asymmetric barriers modulating the self-propulsion can largely enhance the mean drift or even reverse it.

The global effects of Pyragas feedback control Anup Purewal University of Auckland a.purewal@auckland.ac.nz Coauthors: B. Krauskopf C.M. Postlethwaite

Pyragas time-delayed feedback control is designed to stabilize an unstable periodic orbit. The delay in the feedback term is set to the period of the target periodic orbit and, hence, this control scheme is non-invasive. We consider here Pyragas control applied to the widely considered normal form of a subcritical Hopf bifurcation. Previous work focused primarily on the mechanism of stabilization. Here we present a global picture of the dynamics induced by the time-delayed feedback. Furthermore, we discuss the effect of choosing a delay term that is close but not equal to the period of the target orbit. In particular, we consider cases where the delay is either a constant or a linear approximation to the exact period of the target periodic orbit.

### Crop planning with frequently updated weather forecast information

Ramya Rachmawati School of Mathematical and Geospatial Sciences, RMIT University ramya.rachmawati@rmit.edu.au Coauthors: John Hearne, Melih Ozlen

This paper focuses on developing a new crop planning method that utilises weather forecast information. This new method is based on an innovative mathematical model that uses accumulated heat units to determine harvest time. The proposed model is applied in two ways. These are the static and dynamic approaches. The static method, using long-term average temperature and price forecast, means the decision to plant, pull or harvest is made only once, that is at the beginning of a planning horizon. In the dynamic approach, this decision is updated regularly using new information. The dynamic approach gives the result that is slightly higher than that of the static approach. The result of this research is a new model that helps farmers to choose suitable cropping strategies using weather forecasts as an input to maximise their profits.

Robust Bicriteria Optimisation in Transport Optimisation Problems Andrea Raith Department of Engineering Science, The University of Auckland a.raith@auckland.ac.nz Coauthors: Kenneth Kuhn, Marie Schmidt, Anita Schöbel

Many optimisation problems in transport have multiple conflicting objectives. An example of such a bicriteria problem is the routing of aircraft which entails finding a path minimising the distance travelled and exposure to potentially dangerous weather conditions along the way. In transporting hazardous materials one aim is to keep the exposure of population to transported materials in case of an accident low while also keeping travel times at a minimum. Many of these transportation problems feature inherent uncertainty. For example, at the time of planning future weather conditions are unknown and hence weather exposure is uncertain. Likewise, one cannot know travel times precisely, especially in congestion-prone road networks.

One approach to dealing with uncertainty is robust optimisation which is well-studied for single-objective optimisation problems. For example, worst-case robustness identifies solutions that perform best in a worst-case scenario. We extend some of the robustness concepts for single-objective problems to the bicriteria case with one uncertain objective function. Different robustness concepts are developed and analysed. We also propose algorithms to solve the bicriteria robust optimisation problems in the case of finitely many scenarios by extending the problems to related multicriteria optimisation problems. We show computational results for both applications.

Copula-based Approach for Estimating Return Periods of Floods in Victoria, Australia Ummul Fahri Abdul Rauf School of Mathematical and Geospatial Sciences, RMIT University, 3001 Victoria, Australia ummulfahri.abdulrauf@rmit.edu.au Coauthors: Panlop Zeephongsekul

Hydrological phenomena such as precipitation and floods are inherently random by nature. Due to climate change, the issue of floods is receiving increased research attention. Natural disaster protection plans, which include flood mitigation projects, have intensified the studies of rainfall modelling as a key criterion in water resource management. One of the key areas of research is the estimation of recurrence interval or as it is known, the return period. Calculations of the return periods of extreme rainfall or flood event characterized by its severity are essential in many hydrological and water planning projects. Most hydrological events are dependent and separate analysis of each event is not sufficient in assessing overflow water risks or in performing flood analysis. A multivariate approach is generally a more superior method to analyse these events. In this context, the traditional approach of considering the joint distribution of rainfall characteristics using standard bivariate modelling presents some limitations that can be circumvented by using Copula models. In this study, the Archimedean copula distribution with parametric and nonparametric marginal distributions is used to model extreme flood events characterized by rainfall severity and duration. The proposed methodology is applied to the monthly rainfall data from the flood areas of North-eastern and South-western Victoria.

# Coupled Orbital and Thermal Evolution of Major Uranian Satellites

Attique Ur Rehman The University of Auckland attique.ur-rehman@auckland.ac.nz

We have developed a model of the orbital and thermal evolution of the five major Uranian satellites over millions of years. The model consists of detailed ordinary differential equations for the orbital evolution coupled to the one-dimensional heat equation for the thermal evolution. We present preliminary results that show how the different terms in the orbital equations such as the oblateness of Uranus affect the orbital semi-major axis and eccentricity of the satellites.

Modelling the environmental impact of weekly food consumption in Australias different socioeconomic households: an Input-Output approach

Christian Reynolds University of South Australia christian.reynolds@unisa.edu.au Coauthors: Julia Piantadosi; Jon Buckley; Philip Weinstein; John Boland

Australias food supply uses scarce resources such as land, water, oil, and electricity, as shown by the environmental impact related to the Australian agricultural industry (Renouf and Fujita-Dimas 2013). However, there has been little investigation into the environmental impacts of household food consumption.

This paper uses an environmentally extended Input-Output model of the 2003 Australian economy to estimate the environmental impacts of the weekly food consumption for five different socio-economic levels of Australian household.

Using the environmental impact measures of carbon dioxide (Gg), embodied water (m3), waste generated (tonne) and embodied energy (Tj). This paper finds that the environmental impact of food consumed increases with income, even though relative spending on food decreases. This footprint is due to changes in the types of food consumed as incomes increase.

Keywords: environmental impacts; food security; greenhouse gas emissions;

Renouf, M. A. and C. Fujita-Dimas (2013). Application of LCA in Australia agriculture a review. THE 8TH LIFE CYCLE CONFERENCE Pathways to Greening Global Markets, . Novotel Sydney, Manly Pacific ALCAS.

# Respiratory Rhythm Generation in the pre-Botzinger Complex

Kerry-Lyn Roberts University of Sydney K.Roberts@maths.usyd.edu.au Coauthors: Martin Wechselberger, Jonathan Rubin

It is known that a network of synaptically coupled neurons, located in a region of the brain stem known as the pre-Botzinger complex (preBotC), generates respiratory rhythms in mammals. We study a model of two preBoetC neurons by Butera et al (1999) and identify the complex bursting and spiking patterns which arise under variation of two key parameters. By using geometric singular perturbation theory and the technique of averaging over two slow variables, we are able to identify the bifurcations that lead to the transitions between the different regions of activity. We study the unfolding of a codimension 3 bifurcation, which locally organises the observed regions of bursting and spiking behaviour.

Streamflow predictions for industrial applications Melanie Roberts *IBM Research Australia* melanie.roberts@au1.ibm.com Coauthors: Timothy Lynar, Olivia Smith (IBM Research Australia)

Industrial and agricultural activity in the Fitzroy River Basin is commonly influenced by local streamflow conditions. Being a semi-arid environment, this area is characterised by ephemeral streams where high flow events can pass through the system in a matter of hours. For neighbouring businesses, advance knowledge of these events is important for optimising a range of processes.

In this talk we discuss the use of weather forecasts to inform streamflow predictions and hence business activities. The choice of streamflow model must balance the availability of input data against the desired accuracy of the predictions. We illustrate this with an established streamflow model that provides valuable information in certain circumstances yet falls short for other business activities.

# A one-compartment model for body cooling and time of death estimation based only on temperature readings Marianito Rodrigo

University of Wollongong marianito\_rodrigo@uow.edu.au

Time of death estimation is one of the most popular research topics in forensic medicine. Such estimations are important in the investigation of criminal and/or suspicious deaths. In this talk we propose a one-compartment model for body cooling, and which includes the well-known Marshall-Hoare model as a special case. Using Laplace transforms, we derive an explicit formula for the time of death based only on temperature readings. Numerical simulations will be presented.

### Dams Location-Allocation through Economic Water Sharing

Reza Roozbahani School of Mathematical and Geospatial Sciences, RMIT University, Melbourne, Australia RezaRoozbahani@gmail.com Coauthors: S. Schreider, B. Abbasi

Due to population growth, human water needs have a rising trend so that traditional water supply systems such as direct withdrawing surface water cannot satisfy them. Hence, development of new dams can be a solution for increasing available water. Although, dams are helpful for increasing water supply, they could also cause some social and environmental concerns. The selection of dams locations by water authorities always comes along with stakeholders competitions for constructing more dams in their boundaries. In addition, the negative effects of dams on the rivers environment constitute a separate issue which water authorities have to cope with it. This paper introduces a mixed integer program to decide on dam locations, capacity of dams and water allocations to each stockholder. The objective function of model is maximizing the water profits of allocated water to agriculture. Furthermore, the satisfaction of environmental flow is taken into account as a firm constraint in this model. This model is applied for infrastructural development planning in the Sefidrud basin, Iran.

Binary collisions in the planar 3-body problem with vanishing angular momentum Danya Rose University of Sydney danya.rose@sydney.edu.au Coauthors: Holger R. Dullin

Collision orbits, as well as periodic orbits, play an important role as "windows" into the dynamics of the 3-body problem. We will discuss the dynamics of binary collision in the planar 3-body problem with vanishing angular momentum in coordinates globally regularising the binary collisions and examine some families of periodic collision orbits in the case of equal masses that are the "roots" of other families of non-collisional periodic orbits.

**Optimal vaccination strategies** Joshua Ross University of Adelaide joshua.ross@adelaide.edu.au Coauthors: Prof. Matt Keeling and Dr Thomas House

A fundamental problem in public health is how to allocate a limited set of resources such as to have the greatest benefit on the health of the population. This often leads to difficult value judgements about budget allocations. However, one scenario that is directly amenable to mathematical analysis is the optimal allocation of a finite amount of vaccine to a population. It has been shown that an *equalising strategy* (which leaves equal numbers of susceptible individuals in each household) is optimal under certain conditions. Here we consider how robust this strategy is to a range of other scenarios.

### Analytics at Fonterra and beyond. Kevin Ross *Fonterra*

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Fonterra recognises that advanced analytics capability presents an opportunity to develop and sustain a competitive advantage in the global dairy industry. In order to maximize both the value created from milk and shareholder returns, Fonterra needs to employ a combination of descriptive, predictive and prescriptive analytics. We have developed a range of models to support decisions from milk collection to product mix and asset investment. As part of this initiative, we launched the New Zealand Analytics Forum in 2013. This group of Analytics professionals meets regularly to share best practice principles. We see the potential for this to benefit a wide range of industries, and New Zealand as a whole. I will share some of the insights from both Fonterras efforts and the Analytics Forum.

# Canonical Duality Transformation Method for Chaotic Nonlinear Dynamical Systems with Applications

Ning Ruan Federation University Australia n.ruan@ballarat.edu.au Coauthors: David Y. Gao (Federation University Australia/Australian National University)

Duality is a fundamental concept that underlies almost all natural phenomena. The inner beauty in general systems are bound up with duality. Canonical duality theory is a methodological and potentially powerful theory, which can be used not only to model complex systems within a unified framework, but also for solving a large class of nonconvex/nonsmooth/discrete problems in global optimization and nonlinear sciences. Nonconvex phenomena arise naturally in complex systems. It is understood now that the chaotic behaviour in nonlinear dynamics is mainly due to non-convexity of the total potential function. How to identify the global stability and extremality of the critical solutions is a challenge task. In this talk, the speaker will show how to use the canonical duality theory for solving discrete dynamical systems governed by logistic equation. The speaker will first show that by using finite difference and least squares methods, the nonlinear differential equation can be formulated as a non-convex optimization in n-dimensional space. Based on the canonical duality theory, this nonconvex global optimization problem is equivalent to a concave maximization problem which can be solved to obtain global stable solution. Application will be illustrated by a population growth problem of fisheries. The method can also be applied to many NP-hard problems in global optimization and computational science, such as integer programming, network optimization and travelling salesman problem (TSP), etc. This talk should bring some new insights into non-convex analysis, nonlinear dynamics and computational methods. REFERENCES [1] D.Y. Gao, Duality Principles in Nonconvex Systems: Theory, Methods and Applications. Kluwer Academic Publishers, Boston/Dordrecht/London, 2000, xviii+454pp. [2] D.Y. Gao, Canonical duality theory: Unified understanding and generalized solution for global optimization problems, Computers & Chemical Engineering, 33:19641972, 2009. doi:10.1016/j.compchemeng.2009.06.009 [3] Ruan, N. and Gao, DY, Canonical duality approach for non-linear dynamical systems, to appear in IMA J. Appl. Math. doi:10.1093/imamat/hxs067

### Mathematical model of coupled xylem-phloem water transport in leaves

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In the vascular plants water is transported along two main pathways: xylem, where the flow is driven by transpiration, and phloem, where the loading of products of photosynthesis creates an osmotic pressure difference. These two conduits are hydraulically linked. The effects of the hydraulic interaction between xylem and phloem flows have been modelled in some recent studies that have focused on tree stems. However, despite the fact that transpiration, photosynthesis and substantial water exchange occurs in leaves, the models of water transport in leaf vasculature have considered the xylem as an isolated system. We present here a model of water transport in leaves that has been developed to study the mutual influence of xylem and phloem flows. Although, the transpiration and photosynthesis events are not explicitly modelled, the diurnal variations in water flow generated by these processes are simulated using time-dependent changes in xylem and phloem boundary conditions. Two main types of leaf architecture peculiar to monocot and dicot plants are modelled and compared in our study. Our results show improved agreement between theoretical predictions and experimental observations, and confirm the importance of modelling of the xylem-phloem interaction.

# Mitigating Traffic-Related Air Pollution Exposure in Adelaide, South Australia: A Scenario Analysis Using BenMAP

Lisa Schultz

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Coauthors: Belinda Chiera (University of South Australia), John Boland (University of South Australia) and Pushan Shah (Environment Protection Authority).

Traffic-related air pollution exposure has been extensively linked with damaging health effects, including respiratory illness, impaired cognitive function and mortality. The risk of experiencing such health effects can be substantially reduced through the implementation of exposure mitigation strategies, such as reducing traffic volume and congestion as well as applying engine-efficient speed limits.

In this presentation, we apply the Benefits Mapping and Analysis Program (BenMAP) to Adelaide, South Australia, under a number of different scenarios of traffic pollution emissions and reduced exposure. BenMAP is an analysis tool developed by the USA Environment Protection Agency to assess the health and economic benefits associated with particular pollution management strategies. A large group of Australian concentrationresponse functions will be input into BenMAP to quantify health impacts of each scenario considered.

This scenario analysis identifies potential exposure mitigation strategies suitable for Adelaide, SA. Future work will generalise these strategies for implementation in other Australian cities.

The presentation concludes with a discussion of the next stage of the research, focussed on the formulation and solution of an optimisation model for reduced pollution exposure. The model objective is to identify optimal exposure mitigation strategies, and corresponding implementation proportions, to maximise health and economic benefits.

## Rank bounds for Quantized Tensor Train (QTT) Approximations of Invariant Distributions in Chemical Reaction Networks Christoph Schwab SAM ETH Zurich schwab@sam.math.ethz.ch Coauthors: Vladimir Kazeev, ETH Mustafa Khammash, ETH

We consider the QTT approximation of complex chemical reaction networks (CRNs) which arise, among others, in mathematical systems biology. We prove that the stationary distributions of CRNs with deficiency zero in the sense of Feinberg admit numerical approximations in tensor train formatted linear algebra whose complexity scales linearly with respect to the number of molecular species in the network, and at most logarithmically in the desired accuracy of approximation. We consider several widely used models for the kinetics of CRNs, among them classical mass-action kinetics, Michaelis-Menten kinetics and certain generalizations of these. Our result complements recent work on the TT structured numerical simulation of the evolution of system states distribution (given by the Kolmogorov forward equation of the CRN, ie. the Chemical Master Equation). We indicate a perspective of efficient "ab-initio" forward simulation and Bayesian, tensor formatted calibration of CRNs in systems biology. References: Reports 2013-04 and 2013-18 SAM, ETH Zurich. Support: SNF and by ERC AdG 247277

# In Silico Modelling Of Basic Fibroblast Growth Factor Pharmacokinetics In Animal Full Thickness Chronic Wounds

Uzma Sehrish Queensland University of Technology, Brisbane QLD, Australia uzma.sehrish@student.qut.edu.au Coauthors: Associate Professor Dann Mallet; Queensland University of Technology, Brisbane QLD, Australia

The efficacy of a novel therapeutic agent depends on its selective localisation in a target tissue. Such is the desirable feature for topically applied basic Fibroblast growth factor (bFGF/FGF-2) into chronic cutaneous wounds. Due to an increased risk of toxicity, large costs involved, and required length of time, biodistribution studies of FGF-2 in chronic wounds are difficult to carry out in humans. However such data can be obtained in animals. Consequently there is a large volume of clinical literature examining the effects of various empirical doses and treatment regimens of growth factors in patients with chronic ulcers and wounds, but very few examining the kinetics of that growth factor absorption into the treated wounds.

The use of an *in silico* modelling approach, to extract early, fast and relevant pharmacokinetic data on a drug compound, is gaining popularity. Using the data from available studies, we have developed a physiologically based pharmacokinetic model of FGF-2 distribution in animal full thickness skin wounds. The model also incorporates the specific binding of free FGF-2 in the granulation tissue with its cell surface receptors and includes the elimination of free FGF-2 from capillaries and its degradation via internalisation. Our model predicts the tissue concentrations of FGF-2 with sufficient accuracy.

### Title: Distinguishing between mean-field, moment dynamics and stochastic descriptions of birthdeath-movement processes.

Jesse Sharp Queensland University of Technology j3.sharp@qut.edu.au Coauthors: Mat Simpson

Mathematical descriptions of birthdeathmovement processes are often calibrated to measurements from cell biology experiments to quantify tissue growth rates. Here we describe and analyze a discrete model of a birthdeathmovement process applied to a typical two-dimensional cell biology experiment. We present three different descriptions of the system: (i) a standard mean-field description which neglects correlation effects and clustering; (ii) a moment dynamics description which approximately incorporates correlation and clustering effects; and (iii) averaged data from repeated discrete simulations which directly incorporates correlation and clustering effects. Comparing these three descriptions indicates that the mean-field and moment dynamics approaches are valid only for certain parameter regimes, and that both these descriptions fail to make accurate predictions of the system for sufficiently fast birth and death rates where the effects of spatial correlations and clustering are sufficiently strong. Without any method to distinguish between the parameter regimes where these three descriptions are valid, it is possible that either the mean-field or moment dynamics model could be calibrated to experimental data under inappropriate conditions, leading to errors in parameter estimation. In this work we demonstrate that a simple measurement of agent clustering and correlation, based on coordination number data, provides an indirect measure of agent correlation and clustering effects, and can therefore be used to make a distinction between the validity of the different descriptions of the birthdeathmovement process.

Dynamical delivery of volatiles to the outer main belt Philip W. Sharp University of Auckland sharp@math.auckland.ac.nz Coauthors: Kevin R. Grazier Julie C. Castillo-Rogez

We quantify the relative contribution of volatiles supplied from outer Solar System planetesimal reservoirs to large wet asteroids during the first few million years of the Solar System. We simulate the orbital motion of planetesimals originating within different regions of the Solar System, and thus characterized by different chemical inventories, using a very accurate integrator tuned to handle close encounters between planets and planetesimals. The fraction of icy planetesimals crossing the asteroid belt was significant, and our simulations show that planetesimals originating from the Jupiter-Saturn region were orders of magnitude more abundant than those from the Saturn-Uranus and Uranus-Neptune regions when the Jovian planets were embryos. By the time the Jovian planets reached their full masses, the regions contributed similar fractions of planetesimals.

Our result implies that large asteroids like Ceres accreted very little material enriched with clathrate hydrates and low-eutectic volatiles such as methanol, nitrogen and methane ices that current Solar nebula models predict formed in the very low temperatures beyond Saturns orbit.

Helical Flow of Yield-stress Fluids John Shepherd *RMIT University* jshep@rmit.edu.au

Yield-stress fluids have the property that they remain effectively solid until the local stress in their structure reaches a certain 'yield' value, after which they flow as a (generally non- Newtonian) fluid. In the simplest example of such a fluid, the Bingham fluid, the flow, when it occurs, is Newtonian; i.e., the stress versus rate of shearing relationship is linear. Other fluids, such as the Herschel-Bulkley and Casson fluids display more complex stress-rate of shearing relationships upon yielding. We consider helical flow of such fluids in the gap between infinite concentric cylinders, generated by a combination of axial pressure gradient and rotation of the inner cylinder; and the changes in the boundary of any solid-liquid zones with the flow characteristics.

## Integrable quad equations and discrete Painlevé equations Yang Shi The University of Sydney yshi7200@gmail.com Coauthors: Nobutaka Nakazono

In this talk we explain how a class of quad equations known as the ABS (Adler, Bobenko, Suris) equations relate to second-order ordinary discrete nonlinear equations, namely the discrete Painlevé equations. We will demonstrate how the affine Weyl symmetry of a discrete Painlevé equation can be derived from multi-consistant geometry of the ABS equations.

# A Tale of Two Flames - Bistability of Combustion Waves

Harvi Sidhu Applied and Industrial Mathematics Research Group, School of Physical Environmental and Mathematical Sciences, University of New South Wales Canberra, 2600, ACT, Australia h.sidhu@adfa.edu.au

We consider a diffusional-thermal model with two-step competitive reactions for premixed combustion wave propagation in one spatial dimension. Numerical investigation of the flame speed of the system for different fuel types shows the existence of regions where two stable travelling combustion wave solutions co-exist - solutions pertaining to the fast and slow branches. A hysteresis type phenomena exists, and we show that by altering the initial conditions, one can move from one solution branch to the other. Regions of complex pulsating combustion waves were also uncovered.

# Effects of Noise on Nonsmooth Dynamical Systems

David Simpson Massey University d.j.w.simpson@massey.ac.nz Coauthors: Rachel Kuske

Systems of nonsmooth differential equations are increasing being used to model phenomena involving impacts, switches, or other abrupt events. However, many general theorems concerning stochastic perturbations of differential equations do not apply when the equations involve nonsmooth features. In this talk I will describe two basic aspects of how noise affects sliding motion (which is evolution constrained to a surface where the equations are nonsmooth). Over short time frames stochastically perturbed sliding motion can be understood by analysing occupation times through the Feynman-Kac formula. Transitions from sliding to non-sliding are understood by a novel blow-up of phase space centred at the transition point. The results help explain why small noise induces an unexpectedly large increase in the frequency of oscillations in a canonical model of relay control.

# Determining cheese brining times outcomes from MISG 2014

Jo Simpson Fonterra Co-operative Group Limited Jo.Simpson2@fonterra.com

This presentation will look at the outcomes from the MISG 2014 Fonterra problem on determining optimal cheese brining times. For brine salt cheese, blocks of cheese curd are submerged in a brine solution until the desired salt content is achieved (which may take up to three days). The concentration of the brine bath and the brining time are key aspects to ensuring that the product obtains the composition required for the customer specification, with the brining time being the main lever to control the amount of salt absorbed by the cheese. The study group have worked on developing a way to ascertain a more accurate time for the cheese to remain in the brine baths and determine what initial conditions influence this time the greatest.

# How long does it take for aquifer recharge or aquifer discharge processes to reach steady state? Matthew Simpson

Queensland University of Technology matthew.simpsonQqut.edu.au

Groundwater flow models are usually characterized as being either transient flow models or steady state flow models. Given that steady state groundwater flow conditions arise as a long time asymptotic limit of a particular transient response, it is natural for us to seek a finite estimate of the amount of time required for a particular transient flow problem to effectively reach steady state. Here, we use the concept of mean action time (MAT) to address a fundamental question: how long does it take for a groundwater recharge process or discharge processes to effectively reach steady state? This concept relies on identifying a cumulative distribution function, F(t; x), which varies from F(0; x) = 0 to  $F(t; x) \rightarrow 1$  as  $t \rightarrow \infty$ , thereby providing us with a measurement of the progress of the system towards steady state. The MAT corresponds to the mean of the associated probability density function and we demonstrate that this framework provides useful analytical insight by explicitly showing how the MAT depends on the parameters in the model and the geometry of the problem. Additional theoretical results relating to the variance of probability density function, known as the variance of action time (VAT), are also presented. To test our theoretical predictions we include measurements from a laboratory-scale experiment describing flow through a homogeneous porous medium. The laboratory data confirms that the theoretical MAT predictions are in good agreement with measurements from the physical model.

# New Methods for the Accurate and Efficient Evaluation of Molecular Integrals in the B Function Basis

Richard Mikael Slevinsky University of Alberta, Canada rms8@ualberta.ca

Molecular Integrals have a rich history in applied mathematics. They are the computational problem associated with the physical representation of molecular orbitals as a linear combination of atomic orbitals. The computational problem became accessible when Pople elected to use Gaussian orbitals, as the integrals that arise are relatively inexpensive. However, Gaussian orbitals are flawed near the nucleus and in the tails compared with hydrogen-like exponentially decaying orbitals. In the 1980s the B function orbitals, which are special linear combinations of exponentially decaying orbitals, are used for a more accurate representation of atomic orbitals with smaller expansions. Their beautiful Fourier transform allows for explicit analytical expressions for the molecular integrals. The B function approach requires the calculation of millions of one-dimensional semi-infinite highly oscillatory integrals. In this talk, we investigate new robust quadrature methods for their accurate and efficient evaluation.

Zooming in - Multiscale RBF approximation can be locally refined Ian H Sloan The University of New South Wales i.sloan@unsw.edu.au

Because physical phenomena on the earths surface occur on many different length scales, it makes sense when seeking an efficient approximation to start with a crude approximation, and then make a sequence of corrections on finer and finer scales. It also makes sense that eventually fine-scale approximations can be sought locally, rather than globally. In the present talk, describing recent joint work with Q. Thong Le Gia and Holger Wendland, we start with a global multiscale radial basis function (RBF) approximation, based on a sequence of point sets with decreasing mesh norm, and a sequence of (spherical) radial basis functions with proportionally decreasing scale centered at the points. We then show that we can zoom in on a region of particular interest, by carrying out further stages of multiscale refinement on a local region. The process can be continued indefinitely, since the condition numbers of matrices at the different scales remain bounded.

Approaches to diffraction from impedance coated rectangular structures Paul Smith Macquarie University paul.smith@mq.edu.au Coauthors: Anthony Rawlins, Elena Vynogradova

The electromagnetic wave propagation of radio, television and mobile phone signals in cities is influenced by building corners and their surface cladding. An idealised model is provided by the scattering of an E-polarised plane wave from a rectangular cylinder on which an impedance boundary condition is enforced on its cross-sectional boundary. Rawlins recently obtained results on this diffraction problem employing Kellers Geometrical Theory of Diffraction (GTD) and its extensions to deal with multiple diffraction. He utilised the diffraction coefficient for the canonical problem of diffraction by an impedance corner to obtain relatively simple high frequency approximate expressions for the scattered far-field.

In this paper we undertake a numerical study of the same idealised model problem. We employ an integral equation formulation for the unknown surface distribution comprising a single-layer potential and the adjoint double-layer potential. A Nystrom method that is similar to that of Colton and Kress is developed for the numerical solution of this integral equation. The computed scattered far field is compared with the results of Rawlins in order to assess the regime of validity for impedance and frequency. Further validation is provided by recent results of Safonova and Vynogradova obtained by a regularisation method.

# Generalised compact finite difference methods and the spherical dynamo problem.

Andrew Stephan University of Sydney andrews@maths.usyd.edu.au Coauthors: David Ivers

Generalised compact finite difference methods give high-order low-bandwidth finite-difference approximations to linear differential operators and has so far been successfully applied to the spherical kinematic dynamo problem and modelling vortex dipole–wall collisions.

In this talk the modifications necessary to apply the generalised compact method to high order linear differential operators with mixed space/time derivatives is developed in the framework of using (semi) implicit multi-step time-stepping methods. Numerical results for the spherical dynamo problem using these methods will then be compared to established dynamo benchmarks. Application of the methods to spheroidal dynamo modelling will also be briefly discussed.

# Homogeneous viscous flows about corner boundaries Stevan Stojanovic Monash University Stevan.stojanovic@monash.edu Coauthors: Michael Page

A corner boundary condition is present in many situations regarding the dynamics of viscous fluid flow- within a significant subset of these, the steady flow of a homogeneous viscous fluid is appropriate. The singular nature of the viscous flow near corners suggests that numerical techniques may be inaccurate. In this talk, analyticalnumero techniques will be discussed so that these singularities may be resolved more accurately when applying numerical analysis to determine the flow.

# Sintering and stretching in the drawing of microstructured optical fibres

Yvonne Stokes *The University of Adelaide* yvonne.stokes@adelaide.edu.au Coauthors: Peter Buchak, Darren Crowdy

Microstructured optical fibres are made by heating and pulling (drawing) a preform to form a very long fibre. A model will be presented for the drawing process which, in a Lagrangian reference frame, is comprised of a 2D classical Stokes-flow moving-boundary problem for sintering in the cross section under surface tension (the transverse-flow problem), and a 1D axial-stretching problem for reduction in cross-sectional area due to stretching. Remarkably, the fibre geometry is completely described by the scaled boundary length from the 2D transverse-flow solution and the draw ratio, and the balance between surface tension and fibre tension needed to get from preform geometry to fibre geometry is given by a simple constraint. Unlike models previously presented by others, our model is suitable for fibres with arbitrary cross-sectional geometry. The important inverse problem of determining the preform geometry and draw parameters for a desired fibre geometry will be considered, with particular attention given to the difficult issue of invertibility of the 2D transverse-flow problem. Support from the Australian Research Council (DP130101541) and the Leverhulme Trust is gratefully acknowledged.

### Subdiffusion with Nonlinear Particle Interaction

Peter Straka *UNSW Australia* p.straka@unsw.edu.au Coauthors: Sergei Fedotov, University of Manchester

Fractional PDEs and scaling limits of Continuous Time Random Walks have emerged as successful models for subdiffusive dynamics. However, their applicability to subdiffusive systems with particle interaction (e.g. in vivo cells) is limited: Most of the derived fractional equations are linear, whereas interactions typically entail nonlinearities.

We have derived governing fractional PDEs for subdiffusive transport, with any of the following particle interactions: chemical reactions, chemotaxis, adhesion, volume filling and quorum sensing. This provides a means to study the effect a memory effect has on systems with the above particle interactions.

# Detecting stable regions in the global ocean Robyn Stuart University of New South Wales r.stuart@unsw.edu.au Coauthors: Gary Froyland, Erik van Sebille

We use a novel method to investigate the circulation of the surface of the global ocean and to classify the ocean into minimally mixing regions. By using two different input data sets, one that implicitly captures wind effects and one which does not, we can assess how the wind assists transport between different regions of the ocean. We also demonstrate a method to locate quasi-stable rotating regions in the surface of the ocean.

## Modelling Post-Capillary Venules Pavel Sumets University of Auckland psumets@gmail.com Coauthors: John Cater, Richard Clarke

Numerous vascular diseases result from abnormal blood flow. Micro-vessels walls are coated by a brush-like layer, the Endothelial Glycocalyx Layer (EGL), which acts as a buffer between blood and the wall. The EGL is difficult to measure experimentally, hence a mathematical model is potentially useful. Here the EGL is described as a poroelastic medium and flow within the vessel lumen as a viscous Newtonian liquid. A biphasic mixture model is used to approximate the dynamics of the EGL by Brinkmans Equation for the fluid phase, and forced Naviers Equation for the solid phase. A desirable method to solve these linear coupled PDEs is the Boundary Element Method (BEM). However, implementation of BEM is complicated by the presence of integrals over the entire EGL, rather than just boundary integrals. These are due to the momentum transfer terms within the biphasic medium. In order to take advantage of BEM we have therefore developed a technique to convert these area integrals to boundary integrals. This is achieved through the construction of a complementary problem, to which Lorentz Reciprocal relation is applied. As a result, a new efficient and convenient means by which biphasic flow problems can be solved using BEM has been developed.

### Red Blood Cell Classification for Automatic Malaria Diagnosis

Budi Sunarko Flinders University budi.sunarko@flinders.edu.au Coauthors: Simon Williams (Finders University) William R. Presscott (Hydas World Health) Murk J. Bottema (Flinders University) Scott M. Byker (Microscopes International)

Parasite detection is important for the diagnosis of many blood-borne diseases including malaria. As part of a program to develop a fast, accurate and affordable automatic device for diagnosing malaria, a critical step is to automatically classify individual red blood cells (RBC) in a blood smear slide. To automatically recognize malaria parasites in an image, this paper presents a RBC classification study for malaria diagnosis. To diagnose malaria, the threshold-based segmentation is implemented using the Otsu's method succeeded by the distance transform and statistical classifier. The methods are applied to blood images obtained from Hydas World Health. These experimental results show that the classification recognizes malaria parasite with 98.20% accuracy, 98.41% specificity, and 95% sensitivity. Although there was a slightly difference in performs in terms of number of true infected RBC detection between this classification and a human reader at the alpha = 0.05 level (n = 20, p = 0.038), there was no significant difference in case of number of true uninfected RBC detections (n = 20, p = 0.163).

### Unexpected waves in ferrofluid convection

Sergey A. Suslov Swinburne University of Technology ssuslov@swin.edu.au Coauthors: Habibur Rahman, Aleksandra Bozhko

Ferrofluids are modern strongly magneto-polarisable nanofluids flows of which can be non-intrusively controlled by applying an external magnetic field. One of their prospective applications is as a heat carrier in thermal management systems operating in zero-gravity conditions such as those aboard the International Space Station (ISS). The purpose of the analysis that we will present in this talk was the estimation of parametric boundaries of magneto-convection regimes anticipated to occur in an arbitrarily inclined magnetic field in the lead to the corresponding experiments planned onboard ISS. The influence of the field inclination angle was the main purpose of the investigation. It was found that when a field is strictly perpendicular to a ferrofluid layer stationary thermomagnetic patterns develop, but an unexpected form of convection, waves propagating in the direction of concavity of magnetic field lines, have been discovered for non-orthogonally applied fields. In this talk we will endeavour to clarify their physical nature and discuss potential implications for space-based experiments.

# The dynamics of critical points in bounded fluid flows Duncan Sutherland The University of Sydney dsut7890@gmail.com Coauthors: Charlie Macaskill

In an unbounded domain, the well-known forward energy cascade forces energy to large spatial scales. Large numbers of small scale vortices will therefore merge into domain-filling structures. In bounded domains, the walls act as a source of enstrophy which constantly injects small scale vortices that disturb the formation of organised, domain-filling circulation cells. Fayeza Al Sulti and Koji Ohkitani, "Vortex merger and topological changes in two-dimensional turbulence", *Physical Review E*, 86.1016309, 2012 studied vortex merger in unbounded domains by counting the number of elliptic and hyperbolic critical points of vorticity and streamfunction. To identify and classify the critical points Al Sulti and Ohkitani identified points where  $\nabla f = 0$  and classified them using the eigenvalues of the Hessian matrix, but identifying the zeros is difficult numerically. We develop a more reliable and efficient method using a simple nearest neighbour comparison method for finding minima  $(x_i, y_i)$  of  $|\nabla f|^2$  and then compute the Poincaré-Hopf index of  $(x_i, y_i)$  of the vector field  $F = (f_x, f_y)$  to classify it as

a elliptic, hyperbolic, or regular point of the scalar field. Results showing the motion of critical points in time and the variation in the number of critical points over the simulation for bounded geometries will be presented.

#### Reflections on MISG 2014

Troy Farrell and Winston Sweatman *QUT and Massey University* t.farrell@qut.edu.au;w.sweatman@massey.ac.nz

The recent MISG 2014 will have finished two days beforehand. In this session we will outline some of the projects presented and progress made.

Orbits neighbouring equal-mass four-body central configurations Winston Sweatman Massey University, Albany w.sweatman@massey.ac.nz

A collection of masses moving under their mutual gravity can remain in a central configuration indefinitely. Close to a central configuration, orbits can be approximated using perturbations. For four equal masses we identify terms in these perturbations. The dominant perturbation term can be used to determine the evolution of the four-body system beyond the neighbourhood of the quadruple central configuration.

On an oil recovery model with singular well data Kyle Talbot Monash University kyle.talbot@monash.edu Coauthors: Jérôme Droniou

Once the natural pressure in a petroleum reservoir is insufficient for primary extraction methods to remain effective, engineers employ an enhanced oil recovery technique, whereby one injects a solvent into a wellbore and forces out the remaining oil in the reservoir through another well.

Such a miscible displacement problem in porous media flow is modelled by a nonlinearly coupled system of PDEs, the analysis of which is inhibited by discontinuous or potentially unbounded coefficients and singularity of the data. The latter difficulty arises from the relative scale of the wells and the reservoir, and suggests that the action of the wells is most appropriately modelled by Radon measures.

I will motivate the mathematical formulation of the model, highlighting some of the difficulties encountered and the tools we use to handle them, and discuss how we obtain the existence of a solution using a regularisation procedure.

This is joint work with Jérôme Droniou.

# A Model for the BitCoin Block Chain that takes Propagation Delays into Account.

Peter Taylor University of Melbourne p.taylor@ms.unimelb.edu.au Coauthors: Tony Krzesinski

Unlike cash transactions, most electronic transactions require the presence of a trusted authority to verify that the payer has sufficient funding to be able to make the transaction and to adjust the account balances of the payer and payee.

In recent years 'BitCoin' has been proposed as an 'electronic equivalent of cash'. The general idea is that transactions are verified in a coded form in a 'block chain', which is maintained by the community of participants.

Problems can arise when different participants have different versions of the blockchain, something which can happen only when there are propagation delays (at least if all participants are behaving honestly). In this talk I shall present a preliminary model for the behaviour of the blockchain as seen by different participants.

Sensitivity analysis for the Markovian SIR epidemic model Mingmei Teo University of Adelaide mingmei.teo@student.adelaide.edu.au Coauthors: Nigel Bean, Joshua Ross

The Markovian susceptible-infectious-recovered (SIR) epidemic model is commonly used to model the spread of an immunising infection through a population. To use this model in a real-world application, we must estimate the (effective) transmission rate and the recovery rate. This leads a distribution on the parameters of the model. A rigorous study would hence perform a sensitivity analysis when using the model for making predictions. Here we consider whether interval analysis may be used in place of traditional sensitivity analysis, to provide a more efficient and/or rigorous approach to handling parameter uncertainty in predictions. I will present a method for obtaining an approximate cumulative distribution function for the mean final epidemic size and the mean epidemic duration, and provide comparisons of this interval method with standard sensitivity analysis.

Solving PDE boundary control problems as systems of ODEs. Johnny Thew Queensland University of Technology johnny.thew@student.qut.edu.au Coauthors: Dr Pamela Burrage, A. Prof Dann Mallet

In this talk, we consider solving a PDE optimal control problem where the control variable is located on the boundary of the problem domain. A Gauss-Seidel like finite difference scheme will be used to reduce the problem down to a system of ODEs containing a single control variable on the boundary equation. The optimality system can then be set up using Pontryagin's maximum principle before solving the problem. To illustrate this

first discretise then optimise method, a three species model depicting the interactions of basic

fibroblast derived growth factor, a binding agent and the resulting bounded growth factor located in a chronic wound is solved and compared to an alternative

first optimise then discretise method. A discussion regarding the advantages and limitations of the methods will also be conducted.

# Hold or Sell? - The Online Knapsack Problem with Incremental Capacity

Morten Tiedemann University of Goettingen, Germany, Institute for Numerical and Applied Mathematics m.tiedemann@math.uni-goettingen.de Coauthors: Clemens Thielen, Stephan Westphal

We consider an online knapsack problem with incremental capacity. In each time period, a set of items, each with a specific weight and value, is revealed and, without knowledge of future items, it has to be decided which of these items to accept. Additionally, the knapsack capacity is not fully available from the start but increases by a constant amount in each time period. The goal is to maximize the overall value of the accepted items. This setting extends the basic online knapsack problem by introducing a dynamic instead of a static knapsack capacity and is applicable to classic problems such as resource allocation or one-way trading.

In contrast to the basic online knapsack problem, for which no competitive algorithms exist, the setting of incremental capacity facilitates the development of competitive algorithms. We provide a competitive analysis of deterministic and randomized online algorithms for the online knapsack problem with incremental capacity and present lower bounds on the competitive ratio achievable by online algorithms for the problem. Most of these lower bounds match the competitive ratios achieved by our online algorithms exactly or up to a constant factor.

# Sensitivity of edge detection methods for quantifying cell migration assays

Katrina K. Treloar Queensland University of Technology katrina.treloar@connect.qut.edu.au Coauthors: Matthew J. Simpson

Quantitative imaging methods to analyse cell migration assays are not standardised. Here we present a suite of two-dimensional barrier assays describing the collective spreading of an initially-confined population of 3T3 fibroblast cells. To quantify the motility rate we apply two different automatic image detection methods to locate the position of the leading edge of the spreading population after 24, 48 and 72 hours. These results are compared with a manual edge detection method where we systematically vary the detection threshold. Our results indicate that the observed spreading rates are very sensitive to the choice of image analysis tools and we show that a standard measure of cell migration can vary by as much as 25% for the same experimental images depending on the details of the image analysis tools. Using a mathematical model, we provide a physical interpretation of our edge detection algorithms alone do not specify any physical measure, or physical definition, of the leading edge of the spreading population.

Fast Solution of Viscoplastic Flow Problems with Trust-Region SQP Methods Timm Treskatis Department of Mathematics and Statistics, University of Canterbury, Christchurch, New Zealand timm.treskatis@pg.canterbury.ac.nz Coauthors: PhD project supervised by Miguel Moyers-Gonzlez and Chris Price

What do toothpaste, pizza dough and paint have in common? Not only are there numerous blank spots on the map of their underlying theory. Also on the numerical side, existing algorithms for the simulation of such viscoplastic fluids suffer from poor convergence properties or proneness to ill-conditioning. After an introduction to non-Newtonian fluid mechanics, this talk will present first results that have been achieved during the initial stage of this PhD project. This includes an analysis of the governing equations for so-called Bingham and Herschel-Bulkley fluids, from which a new formulation in terms of an optimisation problem can be derived. Furthermore, we will discuss a suitable and very efficient numerical strategy based on well-established concepts in numerical optimisation to achieve fast global convergence, even in spite of singularities in gradients and Hessians. Finally, a summarising comparison to an existing approach reveals strong duality between the old and new formulations. It may be expected that analogue concepts can also be applied to problems of similar structure from other fields.

The slumping and stretching under gravity of viscous fluid cylinders Hayden Tronnolone University of Adelaide hayden.tronnolone@adelaide.edu.au Coauthors: Yvonne Stokes (University of Adelaide) Darren Crowdy (Imperial College London)

A model is presented for the evolution of a cylinder of very viscous fluid containing a number of interior channels. The fluid is modelled as a Stokes flow that deforms due to gravity and surface tension. By using a slenderness approximation and introducing a Lagrangian co-ordinate the problem reduces to a two-dimensional model for the flow in each cross section and a one-dimensional model for the corresponding cross-sectional areas. The model is compared to experimental results and is found to show good agreement. Importantly, it will be shown that both surface tension and gravity are needed to capture the correct qualitative behaviour. The implications of this result on the modelling of microstructured optical fibre fabrication will be discussed.

### A new mode of instability in compressible boundary-layer flows

Adam Tunney University of Auckland a.tunney@auckland.ac.nz Coauthors: Jim Denier (University of Auckland) Trent Mattner (The University of Adelaide) John Cater (University of Auckland)

In low disturbance environments such as aerodynamic flight, the initial growth of disturbances that cause the laminar-turbulent transition process in a boundary layer can be investigated with linear stability theory (LST). A large collection of results using LST are available in the literature, however excluded are the class of boundary-layer flows with region of velocity overshoot. Using a compressible, heated-wall flat-plate boundary layer with a favourable pressure gradient as a prototype, the linear stability of this class of boundary layers is investigated numerically and analytically using LST. A new mode of inviscid instability is found that is localised within the region of velocity overshoot and has two corresponding neutral modes, one of which is explored analytically.

A connection between ideal magnetohydrodynamics and nonlinear elastodynamics Geoff Vasil University of Sydney geoffrey.m.vasil@gmail.com

This talk discusses the dynamics of the Magnetorotational Instability (MRI) in a weakly nonlinear regime. The MRI operates in many astrophysical settings, and in particular, largely powers bright astrophysical accretion disks. I discuss the behavior of this important instability, and derive a reduced analytical model for dynamics near the onset threshold. Surprisingly, the reduced model assumes an identical form for the nonlinear dynamics of a buckling elastic beam. This link elucidates several aspects of both instabilities, and explains the saturation of the MRI even in the absence of magnetic reconnection.

# Canards of Folded Saddle-Node Type

Theodore Vo University of Sydney theodore.vo@sydney.edu.au Coauthors: Martin Wechselberger

The canard phenomenon occurs generically in singular perturbation problems with at least two slow variables. Closely associated with folded singularities, canards are separatrices that partition the flow and organize the dynamics in phase space. Folded node canards and folded saddle canards (and their bifurcations) have been studied extensively in  $\mathbb{R}^3$ . The folded saddle-node (FSN) is the codimension-1 bifurcation that gives rise to folded nodes and folded saddles and has been observed in various applications, such as the forced Van der Pol oscillator and in models of neural excitability. The dynamics of the FSN however, are not completely understood. By combining methods from geometric singular perturbation theory and the theory of dynamic bifurcations (analytic continuation into the plane of complex time), we prove the existence of canards (and faux canards) near the FSN limit and study their properties.

The ghost in our genes - epigenetics is our new frontier Graeme Wake Institute of Natural and Mathematical Sciences, Massey University Albany g.c.wake@massey.ac.nz Coauthors: Andrei Korobeinikov, Centre de Recerce Mathematics, Barcelona, Spain

Evolution is driven by the degree of plasticity, by which living organisms mutate to optimise their fitness. A mechanism is proposed in which the phenotype is modelled on a continuous scale providing a parameter to quantify the phenotype state. Due to their very high replication and mutation rates, RNA viruses can serve as an excellent testing model for verifying hypotheses and addressing questions in evolutionary biology. A simple deterministic mathematical model of the within-host viral dynamics is proposed which assumes a continuous distribution of viral strains in a one-dimensional phenotype space. Simulations show that random mutations combined with competition for a resource results in evolution towards higher Darwinian fitness in the phenotype space. This model incorporates features of the real-life host-virus system such as immune response, antiviral therapy, etc. Properties of the resulting dynamics of these structured populations are given as a means of obtaining underpinning decision support for the evolutionary process involved as a precursor for possible intervention. The steady-state(s) of the steady phenotype distribution involves non-local (integro-differential) equations, similar to those in structured cell-growth models. This work is supported (GCW) by "Gravida" which is acknowledged. Reference: Korobeinikov A and Dempsey C (2013-4, in press). Mathematical Biosciences and Engineering.

The Path of a Light Ray: can we Bend it like Einstein? Steve Walters School of Mathematics and Physics, University of Tasmania Stephen.Walters@utas.edu.au

The path taken by a photon passing by a massive body is determined using Einstein's equations of General Relativity. The exact equations of motion are known for a small number of simple cases. More complicated systems require some form of approximation. This talk will consider approximations for a light ray near a system of two or more bodies, and in the cases when the central body is rotating or not.

### Modeling methane production in ruminants

Yuancheng (James) Wang Massey University, Palmerston North wyc5320gmail.com Coauthors: Dr. Peter Janssen (AgResearch, Grasslands Research Centre)

Dr. Tammy Lynch (Institute of Fundamental Sciences, Massey University)

Dr. David Pacheco (AgResearch, Grasslands Research Centre)

A/Prof. Bruce van-Brunt (Institute of Fundamental Sciences, Massey University)

Acknowledgement New Zealand Agricultural Greenhouse Gas Research Centre for funding

In 2011, ruminant-based (cows, sheep etc.) agriculture contributed \$ NZ17 billion to the export of New Zealand. At the same time, these ruminants produce methane, which constitutes one third of New Zealand's total anthropogenic greenhouse gas emissions. Methane is formed by methanogens (microbes) that metabolise hydrogen in the rumen. The hydrogen is formed during the fermentation of ingested feed by other microbes.

Current mathematical models do not predict methane production as a result of the metabolic activity of the methanogens. A mechanistic mathematical model of the interaction between methanogens and hydrogen could contribute to our understanding and predictions of enteric methane production. This motivated us to develop a model based on biological principles. These include the effect of washout (turnover rate), amount of feed ingested by ruminants and fermentation thermodynamics, which affects hydrogen production and so methane production in the rumen.

In this talk, I will present a model of methane production in the rumen. This model depicts the methanogen growth and hydrogen concentrations in relation to hydrogen input and rumen turnover. I will also discuss biological interpretations of the models and the effects of fermentation thermodynamics.

### Riemann localisation on the sphere

Yuguang Wang The University of New South Wales wangyg85@gmail.com Coauthors: Ian H. Sloan (The University of New South Wales) Robert S. Womersley (The University of New South Wales)

I will talk about the Riemann localisation on the unit sphere in three or more dimensional Euclidean spaces. The Riemann-Lebesgue lemma implies the convergence to zero of the local Fourier convolution on the circle, the convolution with the Dirichlet kernel times an indicator function. We say this convolution has the Riemann localisation property on the circle. The local Fourier convolution has the Riemann localisation property for sufficiently smooth functions on the sphere in the three dimension but not in higher dimensions. We filter the local Fourier convolution has the Riemann localisation on the sphere in any dimension. An application is the local downward continuation problem.

Canard Supersonique... Martin Wechselberger University of Sydney wm@maths.usyd.edu.au

... is a duck that is able to move at sonic speed and beyond!

Impossible? Well, in this talk I shall convince you otherwise. Looking at the gas dynamics of stars under the assumption of spherical symmetry, I will show that transonic events in such systems are canard phenomena –

peculiar solution structures identified in geometric singular perturbation problems. Consequently, *stellar winds* are carried by supersonic ducks, and *canard theory* provides a mathematical framework for this astrophysical phenomenon. So, whenever you have the chance to watch the *southern or northern lights*, remember the superheros behind that scene – transonic canards!

#### Additive Dirichlet Models: Mixtures without data

Simon Williams Flinders University s.williams@flinders.edu.au

To use standard mixture model techniques to model X-ray images the point of arrival of each photon at the film plate would be needed. Instead, all there is to work with is the X-ray itself, which is an intensity map that measures the number of photons arriving over the area covered by a pixel. The absorptive mechanism by which X-ray images are generated means an additive model for the image,  $p(x|\mathbf{V}, \boldsymbol{\Gamma})$ , is required, which can be written in the form

$$p(x|\mathbf{V}, \mathbf{\Gamma}) = \sum_{i=1}^{\infty} \pi_i(\mathbf{V}) p(x|\gamma^i)$$

where  $\pi_i$  are the weights of each mixture and  $p(x|\gamma^i)$  are the distributions of each mixture component. Estimation of the parameters of this model for a given X-ray image using standard expectation maximisation techniques is problematic. However, for bivariate gaussian components the use of the natural parameters allows the calculation of derivatives with respect to each of the parameters for use in gradient-based optimisation techniques.

Predicting the onset of osteoarthritis using discrete dynamical systems Francis G. Woodhouse The University of Western Australia francis.woodhouse@uwa.edu.au Coauthors: Bruce S. Gardiner, David W. Smith

Articular cartilage is a deformable tissue which promotes proper function of diarthroidal joints by providing a low-friction wear-resistant layer on the end of each bone. Over many years, the cartilage in weight-bearing joints such as the knees and hips can thin and degrade, leading to a loss of mobility and function. This condition, termed osteoarthritis, is responsible for billions of dollars in lost labour and millions of joint replacements annually. While cartilage can at least partially self-repair, the absence of a blood supply lengthens the characteristic times of repair processes to months or even years, rendering some lifestyle and activity patterns unsustainable in the long run. To understand the early warning signs for osteoarthritis and the primary damage mechanisms of these patterns, we will idealise cartilage composition into a few key components: collagen, aggrecan and cells. We then model stress-induced damage and repair through a discrete dynamical system coupled to a simple composition-dependent model of cartilage as a porous medium. This allows us to explore long-term cartilage behaviour both analytically, to calculate the feasibility of lifelong homeostasis, and stochastically, to compare the likelihood of osteoarthritis onset across different ages and lifestyles.

# Generation and Control of Solitons using Various Nematic Geometries and Regimes

Annette L. Worthy School of Mathematics & Applied Statistics, University of Wollongong annie@uow.edu.au Coauthors: A.A. Minzoni, Universidad Nacional Autonoma de Mexico, Mexico, N.F. Smyth, University of Edinburgh, Scotland, U.K.

A mathematical treatise of experimental work on the generation and control of optical solitons in the nonlinear nonlocal nematic media will be discussed. It will be shown that having an additional localized voltage to form various suitable regimes causes the director or nematic to have different orientations with the cell. Some geometries that will be studied are the rectangular cross section, circular and elliptical regions.

Using asymptotic methods along with the nematicon being is largely independent of functional form of its profile, it is shown how the beam evolves. It will be revealed that the nematicon sheds radiation whereby the velocity and position decoupled from its width and amplitude oscillations. Further, due to the additional geometrics caused by an external applied voltage to the nematic upon twisting the molecules, the resulting polarized self propagating beams distorts and refracts.

The resulting mathematical analysis is quick and efficient and is shown to give excellent agreement to both experimental work and numerical simulations

# A finite volume scheme with preconditioned Lanczos method for two-dimensional space-fractional reaction-diffusion equaitons $O_{i}^{i} = V_{i}^{i} = V_{i}^{i}$

Qianqian Yang Queensland University of Technology q.yang@qut.edu.au Coauthors: I. Turner, T. Moroney, F. Liu

Fractional differential equations have been increasingly used as a powerful tool to model the non-locality and spatial heterogeneity inherent in many real-world problems. However, a constant challenge faced by researchers in this area is the high computational expense of obtaining numerical solutions of these fractional models, owing to the non-local nature of fractional derivatives. We introduce a finite volume scheme with preconditioned Lanczos method as an attractive and high-efficiency approach for solving two-dimensional space-fractional reaction-diffusion equations. The computational heart of this approach is the efficient computation of a matrix function-vector product f(A)b using the Lanczos method, where A is the matrix representation of the Laplacian obtained from the finite volume method. Preconditioning this matrix function is more challenging than a linear system, but still possible, and we present a preconditioner based on deflation that results in greatly improved efficiency of the method. Our approach is showcased by solving the fractional Fisher equation on a disk, including a validation of the solution and an analysis of the behaviour of the model.

Analysis of a stochastic cell growth model Ali Zaidi Massey University, Auckland a.a.zaidi@massey.ac.nz Coauthors: Bruce van-Brunt (Massey University, Palmerston North), Graeme Wake (Massey University, Auckland)

We modify a cell growth model presented by Zaidi, van Brunt and Wake (2013) to allow for a stochastic growth rate. This modification reveals a second order functional differential equation. The solution is required to be a probability density function for the size of a cell (as measured by DNA content). We study the equation for certain choices of non constant coefficients that correspond to variable dispersion, growth and splitting rates. This choice of coefficients leads to a Bessel type operator, and it is shown that there is a unique probability distribution function that solves the equation. The solution is constructed using the Mellin transform and is given in terms of an infinite series of modified Bessel functions.

Image registration Raziyeh Zarredooghabadi Massey university r.zarre@massey.ac.nz

Image registration is one of the main areas in image processing which gives information about images for instance, how an image has been evolved to another with time. One of the images is referred to as source and the others are referred to as the target images. Image registration involves spatially registering the target image(s) to align with the reference image. It has many applications in medical field, object-class detection, etc. There are some factors in image registration, For example: (1) Transformation which is determined by relating positions in one image to corresponding positions in one or more other images. There are different groups of dependencies in transformation: whole diffeomorphism group and its finite dimensional subgroups, (2) Measure distance, (3) initial guess to start optimization, etc. In this talk I will explain how to align target to the source in finite dimension group.

### Pricing American-style Parisian options

Song-Ping Zhu School of Mathematics and Applied Statistics, University of Wollongong spz@uow.edu.au Coauthors: Nhat Tan Le, Wenting Chen and Xiaoping Lu

In this talk, the pricing of various American-style Parisian options under the Black- Scholes model is discussed first. After pointing out the fundamental difference between American-style "in" and "out" Parisian options, we present an analytic solution for American-style up-and-in Parisian options, which does not explicitly involve a moving boundary as far as the "mother option" is concerned. The solution is worked out after combining the "moving window" technique developed in Zhu and Chen (2013) and the method of images, in order to simplify the solution procedure. Our final solution is written in the form of three double integrals, which can be easily computed numerically.

Optimal damping of servo axes using Gröbner bases Oliver Zirn Pforzheim University, Tiefenbronner Straße 65, 75175 Pforzheim, Germany oliver.zirn@hs-pforzheim.de Coauthors: Lukas Katthän, Volkmar Welker

Structural elasticity and flexible transmission devices are the main bottleneck for the feedback control performance of modern servo drives. In this contribution, we present a new and algebraic approach to the commissioning of servo axes with optimal damping - especially for P/PI-position controllers and the upcoming state space control extensions. The approach is based on the optimization of root loci of the transfer function denominator using Gröbner bases techniques. The results are either explicit formulas for 3rd- and 4th-order systems or descriptions of the optima via roots of univariate polynomials. Based on these results, analytical rules of thumb for servo P/PI-controller gains can be derived that allow efficient servo axis setup as well as (semi-) automatic controller commissioning. The benefit of the results is demonstrated by several industrial examples (machine tools, robots, miniature drive systems). The cooperation between the Discrete Mathematics research group of the Marburg University and the Pforzheim University of Applied Sciences was funded by the german research foundation as well as by three industrial partners.

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